On the Chaotic Representation Property of Certain Families of Martingales and Applications

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We start by giving a brief historical review of what is known about the chaotic representation property (CRP) in terms of normal random measures and independent random measures. In this context, there arises the natural question whether the CRP can be extended to more general orthogonal random measures. Thereafter we move on to families $\mathscr{X}$ of square integrable martingales, the main object of our investigation. Under the assumption that their predictable covariations are deterministic, we shall introduce iterated integrals and state their properties. On this basis, the definition of the CRP will be given. An important property of $\mathscr{X}$ is its so-called *compensated-covariation stability*. It turns out that under this additional condition on $\mathscr{X}$, every stochastic integral of monomials in martingales from $\mathscr{X}$ with respect to martingales from $\mathscr{X}$ again belongs to the space of elementary iterated integrals $\mathscr{J}_e$. This seems an important property, one of the main ingredients for the further developments. We continue by providing sufficient conditions on $\mathscr{X}$ for possessing the CRP. As a first illustration of this general result, then we discuss Gaussian families of (local) martingales and independent families of compensated Poisson processes. Thereafter we focus our attention to Lévy processes L: We consider certain families $\mathscr{X}$ of square integrable martingales with respect to the Lévy filtration $\mathbb{F}^L$ and we provide *necessary and suffficient* conditions for the CRP of $\mathscr{X}$. As a special case, under certain conditions on the Lévy measure of L, this includes the so-called Teugels martingales studied by Nualart & Schoutens (2000). We close by discussing some more examples and potential applications of the stated results on the CRP.