Limit theorems for fixed points of Ewens random permutations

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Introduction. Let σ_n be a random permutation sampled from symmetric group S_n according to the Ewens sampling formula:

$$\mathbb{P}(\sigma_n = \pi) = \frac{\theta^{\mathrm{c}(\pi)}}{\theta(\theta+1)\dots(\theta+n-1)},\ \pi \in \mathrm{S}_n,$$

where $\mathrm{c}(\pi)$ denotes the number of cycles in π. The case of $\theta = 1$ corresponds to the uniform distribution on S_n. Consider the sequence of point processes $(P_n, n \geq 1)$ on $[0,1]$ defined by

$$P_n = \sum_{\substack{i=1,\dots,n:\\ \sigma_n(i)=i}} \delta_{\frac{i}{n}},$$

where δ_x denotes Dirac measure concentrated at x. Specifically, $P_n([0,1])$ is the number of fixed points of σ_n. It is known [1] that the limiting distribution of $P_n([0,1])$ as $n \to \infty$ is Poisson with rate θ.

Main results. The core result is the vague convergence in distribution of $(P_n, n \geq 1)$ to the θ-rate homogeneous Poisson point process N on $[0,1]$ as $n \to \infty$. This is equivalent [2] to distributional convergence of the corresponding cumulative processes in the Skorokhod J_1 topology. Using the continuous mapping theorem for functionals on the space of point measures, convergence in distribution for various statistics of fixed points of σ_n can also be proven. This work covers limiting distributions of the smallest and largest fixed points, sum of fixed points, and the smallest and largest spacings between fixed points, which are given explicitly.

The talk is based on the joint work with Andrii Ilienko.

References

[1] R.Arratia, D.Barbour and S.Tavaré, *Logarithmic combinatorial structures: Aprobabilistic approach*, ser. EMS Monogr. Zürich: European Mathematical Society (EMS), 2003.

[2] O. Kallenberg, *Random Measures, Theory and Applications*, ser. Probability Theory and Stochastic Modelling, Springer International Publishing Switzerland, 2017.

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