吉林大学 JILIN UNIVERSITY

数学学院

National Academy of Sciences of Ukraine
INSTITUTE OF MATHEMATICS

International

Summer school

for students and young researchers

Modern problems in Stochastic Processes

July 31 – August 11, 2023

Welcome to the "Modern Problems in Stochastic Processes" Summer School!

The primary goal of the summer school is to shed light on the modern problems encountered in various branches of stochastic processes theory. Through a series of comprehensive lectures and discussions, we aim to navigate through contemporary research and applications within this field. The lectures are suitable for undergraduate students, master students, young researchers and everyone who is interested in this field.

The summer school provides an excellent platform for creating connections and strengthening international collaborations.

This event is divided into two parts. The first part will be conducted online via Zoom. The second one takes place at Jilin University (China).

The online part of the summer school will allow participants to join remotely from anywhere in the world, providing convenient access to high-quality lectures.

All lectures will be recorded and will be available on the YouTube channel https://www.youtube.com/@theoryofstochasticprocesses

Organization committee

Prof. Dorogovtsev Andrey A., Department of Theory of Random Processes, Institute of Mathematics, National Academy of Science of Ukraine

Prof. Yuecai Han, Department of Probability Statistics and Data Science, School of Mathematics, Jilin University, China.

Prof. Xia Chen, Department of Mathematics, University of Tennessee, Knoxville

Ph.D. Hlyniana Kateryna School of Mathematics, Jilin University, (China), Institute of Mathematics, National Academy of Science of Ukraine.

Sponsors: Jilin University, School of Mathematics.

List of lectures

- **Dorogovtsev Andrey A.**
  National Academy of Science of Ukraine, Institute of Mathematics
  "*Ergodic theory for equations with interaction*"
- **Kinderknecht Yana**
  Kassel University (Germany), Institute of Mathematics
  "Semigroups generated by integro-differential operators in Stochastics and Mathematical Physics"
- **Naoufel Salhi**
  University of Tunis El Manar
  "Five lectures about self-intersection local times"
- **Ryabov Gerogii**
  National Academy of Science of Ukraine, Institute of Mathematics
  "*Stochastic flows and random dynamical systems*"

Schedule, week 1.

Online lectures

Date	Time	Lecture	Location
Monday, July 31	14:00-15:00	Ryabov G. Lecture 1. Stochastic flows of solutions of smooth stochastic differential equations.	zoom
	15:30-16:30	Dorogovtsev A.A. Lecture 1. Wasserstein space. Compactness criteria	zoom
Tuesday, August 1	14:00-15:00	Ryabov G. Lecture 2. Smooth random dynamical systems. Perfection theorems.	zoom
	15:30-16:30	Dorogovtsev A.A. Lecture 2. Equations with interaction. Existence and properties of solutions	zoom
Wednesday, August 2	14:00-15:00	Ryabov G. Lecture 3. Coalescing stochastic flows on the real line.	zoom
	15:30-16:30	Dorogovtsev A.A. Lecture 3. Stationary solutions to ordinary stochastic differential equations	zoom
Thursday, August 3	14:00-15:00	Ryabov G. Lecture 4. Duality for coalescing stochastic flows on the real line.	zoom
	15:30-16:30	Dorogovtsev A.A. Lecture 4. Shift-compactness as a substitution for stationarity	zoom
Friday, August 4	14:00-15:00	Ryabov G. Lecture 5. Coalescing stochastic flows on metric graphs	zoom
	15:30-16:30	Dorogovtsev A.A. Lecture 5. Recent results: intermittency of the density, winding numbers	zoom

Schedule, week 2.

Lectures in Jilin University

Date	Time	Lecture	Location
Monday, August 7	9:00-10:30	Kinderknecht Y. Lecture 1. Operator semigroups, evolution equations, and Markov processes	Jilin University, Mathematical Building, Conference Hall 1
	14:00-15:00	Naoufel S. Lecture 1. Local times of Gaussian integrators.	Jilin University, Mathematical Building, Conference Hall 1
Tuesday, August 8	9:00-10:30	Kinderknecht Y. Lecture 2. Feller semigroups / Feller processes	Jilin University, Mathematical Building, Conference Hall 1
	14:00-15:00	Naoufel S. Lecture 2. Self-intersection local times of Gaussian integrators.	Jilin University, Mathematical Building, Conference Hall 1
Wednesday, August 9	9:00-10:30	Kinderknecht Y. Lecture 3. Construction and approximation of operator semigroups	Jilin University, Mathematical Building, Conference Hall 1
	14:00-15:00	Naoufel S. Lecture 3. Self-intersection local times, polymers, and large deviations.	Jilin University, Mathematical Building, Conference Hall 1
Thursday, August 10	9:00-10:30	Kinderknecht Y. Lecture 4. Chernoff approximation of evolution semigroups	Jilin University, Mathematical Building, Conference Hall 1
	14:00-15:00	Naoufel S. Lecture 4. Self-intersection local times as a generalized function.	Jilin University, Mathematical Building, Conference Hall 1
Friday, August 11	9:00-10:30	Kinderknecht Y. Lecture 5. Stochastic processes and evolution equations in the models of anomalous diffusion	Jilin University, Mathematical Building, Conference Hall 1
	14:00-15:00	Naoufel S. Lecture 5. Generalized double intersection local times of the Brownian motion.	Jilin University, Mathematical Building, Conference Hall 1

Lectures

Dorogovtsev Andrey A.

Professor Andrey Dorogovtsev is a corresponding member of the National Academy of Sciences of Ukraine, the head of the Department of Theory of Random Process of the Institute of Mathematics of the National Academy of Sciences of Ukraine, mainly engaged in the research of probability theory and related fields. Under his supervision, 15 students defended their Ph.D. theses, and 4 received a habilitation degree.

His research interests cover different areas such as probability theory, stochastic processes, Malliavin calculus, stochastic differential equations, random operators, measure-valued processes, and stochastic flows. He has a teaching experience in different universities both in Ukraine and outside. He was invited many times as visiting professor to universities in the USA, China, Netherlands, Tunisia, Turkey, Spain, UK. In 2003 he received the State Prize of Ukraine in Science and Technology. Professor Andrey Dorogovtsev is a member of the editorial board of Theory of Stochastic Processes, Ukraine Mathematical Journal, Journal of Stochastic Analysis.

Professor Andrey Dorogovtsev will give a mini-course devoted to ergodic theory for equations with interaction.

Lecture course: Ergodic theory for equations with interaction

Lecture 1. Wasserstein space. Compactness criteria.

Lecture 2. Equations with interaction. Existence and properties of solutions.

Lecture 3. Stationary solutions to ordinary stochastic differential equations.

Lecture 4. Shift-compactness as a substitution for stationarity.

Lecture 5. Recent results: intermittency of the density, winding numbers.

Georgii Riabov

The lecturer is a Fellow researcher at the Department of Theory of Random Process, Institute of Mathematics, National Academy of Science of Ukraine, with a Ph.D. degree. His primary area of research lies in stochastic analysis, specifically focusing on topics such as Gaussian measures, measures on infinite-dimensional spaces, and infinite-dimensional analysis.

He has publications in various international journals and participated in a lot of international conferences. His research has garnered recognition, including the Supreme Council of Ukraine Award for talented fundamental and applied research scientists in 2018.

During summer school, he will give a mini-course devoted to stochastic flows and random dynamical systems.

Lecture course: Stochastic flows and random dynamical systems

Lecture 1. Stochastic flows of solutions of smooth stochastic differential equations.

In this lecture we will consider stochastic differential equations in R^d with sufficiently regular coefficients. It will be proved that such equations can be solved simultaneously for all initial conditions and that the obtained solutions constitute the smooth stochastic flow of homeomorphisms of R^d.

Lecture 2. Smooth random dynamical systems. Perfection theorems.

We will give the definition of a random dynamical system and related notions. Random dynamical systems that correspond to stochastic flows of solutions of smooth stochastic differential equations will be constructed via perfection theorems.

Lecture 3. Coalescing stochastic flows on the real line.

We will consider stochastic flows on the real line, for which a coalescence phenomenon occurs. Such flows are not produced by solutions to smooth stochastic differential equations. Sufficient conditions for coalescing flow to exist will be formulated in terms of its finite-point motions.

Lecture 4. Duality for coalescing stochastic flows on the real line.

We will study dual flow to a coalescing stochastic flow on the real line. This is a flow that moves backward in time and whose trajectories do not cross the trajectories of the initial stochastic flow. Sufficient conditions for dual flow to exist will be given.

Its distribution will be characterized. Obtained results will be applied to analyze the distribution of clusters in Arratia flows with drift.

Lecture 5. Coalescing stochastic flows on metric graphs.

A new approach to the construction of coalescing stochastic flows on metric graphs will be developed. An analogue of the Arratia flow on a metric graph will be constructed.

Yana Kinderknecht (Butko)

The lecturer is a Substitute Professor for Stochastics at the Kassel University, Institute of Mathematics in Germany. She has substantial experience teaching at several universities, including the Technical University Braunschweig, Saarland University, and Bauman Moscow State Technical University.

The lecturer has a lot of publications in high-rank international journals. Her research contributions cover topics such as time-fractional evolutional equations, Feller semigroups, and Chernoff approximation.

Yana Kinderknecht was granted the Mobility funds of G-RISC for outstanding researchers from Germany teaching compact interdisciplinary classes. In 2011, she was awarded the Prize for the Best Teacher for the academic year 2010/2011 at Bauman Moscow State Technical University.

During summer school, she will give a mini-course devoted to semigroups generated by integro-differential operators in Stochastics and Mathematical Physics.

Lecture course: Semigroups generated by integro-differential operators in Stochastics and Mathematical Physics

PD Dr. Yana Kinderknecht (Butko).

Lecture 1. Operator semigroups, evolution equations, and Markov processes

We introduce the objects mentioned in the title, outline the interplay between them and present some cornerstones of the Theory of Operator Semigroups.

Lecture 2. Feller semigroups / Feller processes

We discuss an important subclass of Markov processes: Feller processes and in particular Lévy processes. We establish properties of the corresponding semigroups and integrodifferential evolution equations (e.g., equations with fractional Laplacians and relativistic Hamiltonians), find connections between convolution semigroups of measures, infinitely divisible distributions, Lévy processes, and continuous negative definite functions.

Lecture 3. Construction and approximation of operator semigroups

Different approaches to construct/approximate an operator semigroup will be discussed. We start with some standard procedures and discuss the perturbation techniques, Bernstein functions, and subordination of operator semigroups / stochastic processes in the sense of Bochner.

Lecture 4. Chernoff approximation of evolution semigroups

We present the Chernoff theorem and its corollaries, construct some Chernoff approximations for Feller semigroups (in particular, for Feller diffusions) and obtain the Feynman– Kac formula for a particle in external potentials. We present Chernoff approximations for operator semigroups with additively and multiplicatively perturbed generators. We discuss the relations of Chernoff approximations of operator semigroups with numerical schemes for PDEs and SDEs.

Lecture 5. Stochastic processes and evolution equations in the models of anomalous diffusion

We present a general model of continuous time random walks (CTRWs), leading to different types of diffusion (standard diffusion, subdiffusion, superdiffusion, fractional diffusion) and obtain governing equations for probability density functions of the processes being the scaling limits of CTRWs. In the regime of standard diffusion, one obtains the standard diffusion equation. In some particular cases of other regimes of CTRWs, the governing equations are actually time- or/and space-fractional diffusion equations. We discuss a large class of generalized time-fractional evolution equations, the subordination structure of their solutions (and hence their relation to semigroups and Markov processes), and different classes of underlying stochastic processes. In particular, we present Feymnan-Kac formulae for solutions of such equations on the base of Markov processes time-changed by inverse subordinators, and on the base of randomly scaled Gaussian processes (such as Generalized Grey Brownian Motion (GGBM)).

Naoufel Salhi

Naoufel Salhi is currently an Associate Professor University of Tunis El Manar and has a Ph.D. in mathematics.

His professional experience includes teaching at various institutions. Since February 2023, he has been teaching at the Higher Institute of Applied Sciences and Technology in Kairouan. Prior to that, from September 2012 to January 2023, he taught at the Preparatory Institute for Engineering Studies in Nabeul.

His research interests cover various areas of stochastic analysis, including Brownian motion, Gaussian processes, stochastic integrals, Gaussian measures in Banach spaces, chaos decomposition, local times, and self-intersection local times.

During summer school he will give five lectures about self-intersection local times of Gaussian processes.

Lecture course: Five lectures about self-intersection local times

Naoufel Salhi

Lecture 1. Local times of Gaussian integrators.

local time, occupation formula, criteria for the existence of local time, the case of the Brownian motion, Gaussian integrators, examples, some properties, the existence of local time of Gaussian integrators, some representations.

Lecture 2. Self-intersection local times of Gaussian integrators.

multiple points of Brownian motion, self-intersection local times, criteria for existence, the case of the Brownian motion, renormalization, self-intersection local times of Gaussian integrators, some representations.

Lecture 3. Self-intersection local times, polymers, and large deviations.

polymers, polymer models, end-to-end distance, Flory exponent, self-intersection local times of planar Brownian motion conditioned on the end value, some elements from large deviation theory.

Lecture 4. Self-intersection local times as a generalized function.

double intersection local times of Brownian motion in dimension $\geqslant 4$, Sobolev spaces over Wiener space, Itô-Wiener expansion, representation of positive generalized Wiener function by a measure on the Wiener space, integral formula, capacity of the support of the measure, sufficient condition for the existence of self-intersection local times in some Sobolev space.

Lecture 5. Generalized double intersection local times of the Brownian motion.

finite-dimensional marginals of the measure associated with generalized double intersection local times of Brownian motion, the existence of moments, finite-dimensional marginals of the Wiener measure, quadratic Wasserstein distance, relative entropy, Talagrand inequality.