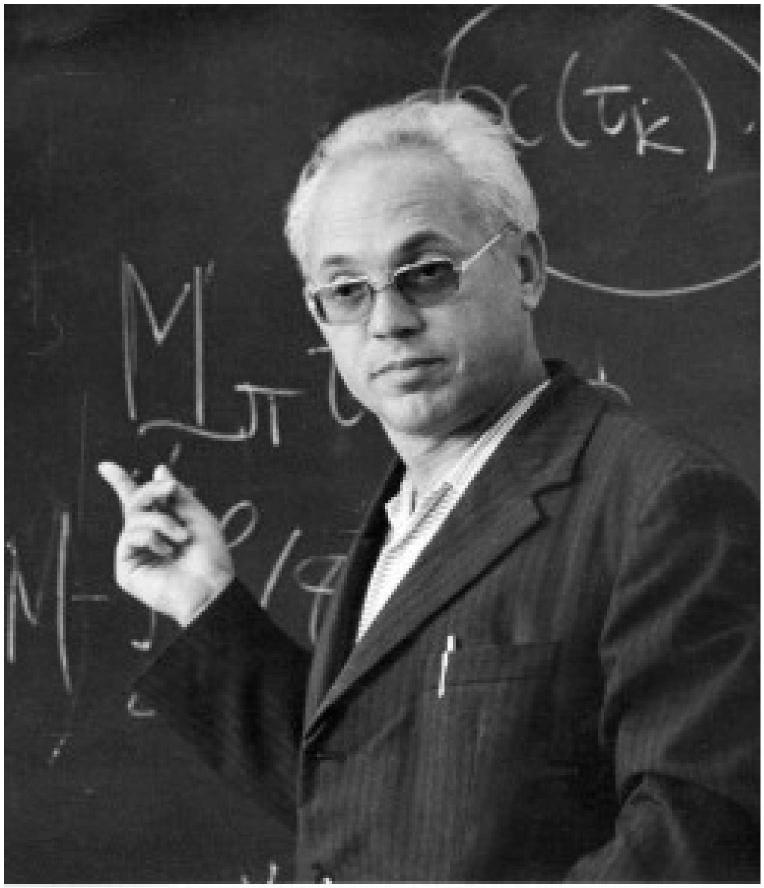

ABSTRACTS

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Contents

Analytic and stochastic description of Brownian motions on star-like graphs

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Let S be a star-like graph, which is a union of finite number of rays with a common vertex $\mathfrak{o}$. We provide a complete description of all Feller processes on S that behave like a Brownian motion on each ray until hitting $\mathfrak{o}$. These processes are described from both an analytical and a stochastic point of view.

The description includes:

(a) construction of generators of Markov semigroups on S with Wentzell-Feller boundary conditions at $\mathfrak{o}$ that describe a mixture of reflection, killing, slowing a time, and jump exit at $\mathfrak{o}$ with possibly infinite intensity;

(b) calculating the explicit form of a resolvent of the process;

(c) representation of the process as a measurable function of finite number of Wiener processes and subordinators.

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Some recent results on Brownian motion with sticky-reflecting boundary diffusion

Marie Bormann

Let Ω be a smooth compact connected Riemannian manifold of dimension $d \geq 2$ with smooth connected boundary $\partial\Omega$. We consider the semigroup on $C(\Omega)$ induced from the Feller generator $(\mathcal{D}(L), L)$ given by where $\frac{\partial f}{\partial N}$ is the outer normal derivative, Δ^{τ} is the Laplace-Beltrami operator on $\partial\Omega$, $\delta \geq 0$ and $\gamma > 0$. The induced Markov process is a diffusion on Ω which performs Brownian motion in the interior while its boundary behaviour consists of Brownian motion with speed according to δ along the boundary as well as sticky reflection back into the interior with intensity γ. Stickiness refers to the fact that the process has a positive occupation time at the boundary. The case $\delta = 0$ corresponds to pure sticky reflection without diffusion along the boundary. This boundary behaviour fundamentally differs from reflection or killing at the boundary.

While the study of diffusions with boundary behaviour involving stickiness and boundary diffusion goes back to [10], in this talk we aim at giving a brief overview on some recent results as well as open questions with regard to the study of Brownian motion with sticky-reflecting boundary diffusion: These recent results involve a rigourous construction via Dirichlet forms ([7]), a number of results on Poincaré and logarithmic Sobolev inequalities ([1, 4, 8, 11]) and Cheeger-type inequalities ([2]), as well as results on large deviations ([5]) and the study of the associated Fokker-Planck equation as gradient descent dynamics ([3, 6]). While certainly of independent interest, Brownian motion is also naturally connected to systems of particles with reversible coalescing-fragmentating interactions ([9]).

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Dynamical Foundation of the Fractional Brownian Motion and Related Conditionally Gaussian Models of Anomalous Diffusion

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Anomalous diffusion is an established phenomenon but still a theoretical challenge in non-equilibrium statistical mechanics. Physical models are built incrementally, and the most recent and most general family is based on the fractional Brownian motion (fBm) with a random diffusion coefficient (superstatistical fBm) together with a time-dependent random Hurst parameter. We provide a dynamical foundation for such general family of models. We consider a dynamical system describing the motion of a test-particle surrounded by N Brownian particles with different masses. This dynamic is governed by underdamped Langevin equations. Physical principles of conservation of momentum and energy are met. We prove that, in the limit $N \to \infty$, the test-particle diffuses in time according to a quite general (non-Markovian) Gaussian process whose covariance function is determined by the distribution of the masses of the surround-particles. In particular, with proper choices of the distribution of the masses of the surround-particles, we obtain fBm together with a number of other special cases of interest in modelling anomalous diffusion including time-dependent anomalous exponent. Furthermore, when the ensemble heterogeneity of the surround-particles embodying the environment becomes non-uniform and joins with the individual inhomogeneity of the test-particles, we show that, in the limit $N \to \infty$, the test-particle diffuses in time according to a quite general conditionally Gaussian process that can be calibrated into a fBm with random diffusion coefficient and random time-dependent Hurst parameter. We conclude our study by reporting the generalised Kolmogorov–Fokker–Planck equations associated to these highly general processes.

This work has been accomplished during my work at Kassel University in collaboration with Christian Bender, Gianni Pagnini and Mirko D'Ovidio.

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A Limit Theorem for Triangular Arrays Generated by Potts Model

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Abstract

The main goal of this presentation is to demonstrate that the limit theorems for the conditional Gibbs measures of Potts model with $(2m+1)$ spins on the Cayley tree of order $2m$ can be considered as examples of multitype branching processes.

In [1] the authors consider the Ising model on the Cayley tree of second order and demonstrate that the limit Gibbs measures are best studied as examples of a two–type branching process.

Let $\Phi = \{-m, -m+1, \dots, -1, 0, 1, \dots, m\}$ be the set of spin values and $\Gamma_+^{2m+1} = (V, E)$ be the Cayley tree of order $2m$, i.e. an infinite graph without cycles with $(2m+2)$ edges issuing from each vertex except for the root x_0, which has only $(2m+1)$ edges. Two vertices $x, y \in V$ are called nearest neighbors if there exists an edge $l \in E$ connecting them, denoted by $l = \langle x, y \rangle$. The distance $d(x, y)$ is the number of edges in the shortest path between x and y.

For a semi–infinite Cayley tree with root $x_0 \in V$ we set

$$W_n = \{x \in V \mid d(x, x_0) = n\}, \qquad V_n = \bigcup_{k=0}^{n} W_k.$$

Assume that at each point of V there is a spin which takes values in Φ. The configuration space is

$$\Omega = \{\omega = \{\omega(x) : x \in V\} \mid \omega(x) \in \Phi\}.$$

We consider the Potts model with $(2m+1)$ spins defined on Γ_+^{2m+1} with Hamiltonian

$$H(\omega) = -J \sum_{x,y \in V} \delta_{\omega(x)\omega(y)}.$$

Let Ω_n be the set of configurations on V_n and ω_n a fixed configuration on $V \setminus V_n$. The conditional Hamiltonian is defined by

$$H(\omega_n \mid \omega_n) = -J \sum_{\langle x,y\rangle,\, x,y \in V_n} \delta_{\omega(x)\omega(y)} - J \sum_{\langle x,y\rangle,\, x \in V_n,\, y \notin V_n} \delta_{\omega(x)\omega_n(y)}.$$

The conditional Gibbs measure $\mu_n(\cdot \mid \omega_n)$ on V_n is given by

$$\mu_n(\omega_n \mid \omega_n) = Z_n^{-1}(\omega_n) \exp\bigl(-\beta H(\omega_n \mid \omega_n)\bigr), \tag{1}$$

where $\beta = (k_B T)^{-1}$ and

$$Z_n(\omega_n) = \sum_{\omega_n \in \Omega_n} \exp\bigl(-\beta H(\omega_n \mid \omega_n)\bigr).$$

We show that the limit theorems for these conditional Gibbs measures can be considered as examples of $(2m+1)$–type branching processes. Applying limit theorems for multitype branching processes [3], we prove the existence of a critical value $\tilde{\beta}_c$ such that for $\beta < \tilde{\beta}_c$ a central limit theorem holds for the corresponding triangular arrays.

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Some remarks on the asymptotic of the critical Galton-Watson branching process with infinite variance

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Let $\{Z(n),\ n \geq 0\}$ be a branching Galton-Watson process starting with a single particle $(Z(0) = 1)$ in which particles leave behind a random number of immediate descendants according to a probability law, having a generating function

$$f(s) = \sum_{k=0}^{\infty} p_k s^k, \ \ 0 \leq s \leq 1,$$

where $p_k = P(Z(1) = k)$. We set $f_0(s) = s$ and introduce iterations of the function $f(s)$, defined by the equality $f_n(s) = f(f_{n-1}(s))$, $n = 1, 2, ...$ The process $\{Z(n),\ n \geq 0\}$ is called critical if $f'(1) = 1$. (For the definition and properties of random branching processes, see [2-3]).
H. Kesten, P. Ney and F. Spitzer [1] proved a theorem that states if the condition

$$f'(1) = 1,\ \sigma^2 = f''(1) < \infty$$

hold, then

$$\lim_{n\to\infty} \frac{1}{n}\left[\frac{1}{1 - f_n(s)} - \frac{1}{1-s}\right] = \frac{\sigma^2}{2}$$

uniformly in $s \in [0, 1)$.
We introduce condition **A**: the generating probability function $f(s)$ has the form

$$f(s) = s + (1-s)^{1+\alpha}\phi(1-s)$$

where $\alpha \in (0, 1]$, $\phi(s)$ is such a function that $\phi(s) \to C$ as $s \to 0$, where C is fixed number.
It is obvious that if condition A holds, then $f''(1) = \infty$ for $\alpha \in (0, 1)$.
We will introduce next the notation $\phi_n = \phi(1 - f_n(0))$, $\gamma_n = |\phi_n - C|$, $\Upsilon_n = \sum_{i=0}^{n} \gamma_i$,
$\psi_n(s) \equiv \phi_n(1 - f_n(s))$, $\delta_n = \sup\limits_{0\leq s\leq 1} |\psi_n(s) - C|$, $\Delta_n = \sum_{i=0}^{n-1} \delta_i$.
We formulate our main results.

Theorem 1. Let condition **A** hold. Then as $n \to \infty$

$$1 - f_n(0) \sim (C\alpha n)^{-1/\alpha}\left[1 + \mathrm{O}\left(\frac{\max(\Upsilon_n, \log n)}{n}\right)\right].$$

Theorem 2. Let condition **A** hold and $\delta_n \to 0$ as $n \to \infty$. Then

$$\lim_{n\to\infty} \frac{1}{n}\left[\frac{1}{(1 - f_n(s))^{\alpha}} - \frac{1}{(1-s)^{\alpha}}\right] = C\alpha$$

uniformly for $s \in [0,1)$.

Corollary. Let are conditions of theorem 2 hold. Then uniformly for $s \in [0,1)$ as $n \to \infty$

$$\frac{1}{\left(1-f_n\left(s\right)\right)^{\alpha}} = \frac{1}{\left(1-s\right)^{\alpha}} + \alpha C n + \mathrm{O}\left(\max\left(\Delta_n, \log n\right)\right).$$

Moreover

$$\left(1-f_n\left(s\right)\right)^{\alpha} = \frac{1+\varepsilon\left(s,n\right)}{\left(1-s\right)^{-\alpha} + C\alpha n},$$

where $\Delta_n = \sum_{i=1}^{n} \delta_n$, $\sup\limits_{s} \left|\varepsilon\left(s,n\right)\right| = \mathrm{O}\left(\frac{\max(\Delta_n, \log n)}{n}\right)$.

We should be noted that in case $\phi\left(s\right) \equiv C$, where $C > 0$ is constant, we have $\phi_n \equiv C$, $\Upsilon_n \equiv 0$, $\Delta_n \equiv 0$ and our results are consistent with the results from [4].

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Winding number of a Gaussian random field on a closed planar curve

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Definition 1 *The winding number $\gamma(\xi, L)$ of the continuous vector field $\xi : \mathbb{R}^2 \to \mathbb{R}^2$ along the continuous curve $L : [0,1] \to \mathbb{R}^2$ is the number $\frac{1}{2\pi}(\phi(1) - \phi(0))$, where ϕ is the continuous branch of the angular function of ξ.*

The main object of study is the Gaussian stationary random field $\xi : R^2 \supseteq U \to R^2$ with covariance function

$$\mathbb{E}\xi_i(u)\xi_i(v) = e^{-||u-v||^2},$$

namely, the winding number of ξ on the boundary of compact set $B \subset \mathbb{R}^2$ (further denoted as $\gamma(\xi, B)$). We assume that the boundary of B is a closed curve with continuous parametrization $\psi : [0,1] \mapsto \mathbb{R}^2$. The winding number can be calculated as follows [1]:

$$\gamma(\xi, B) = \sum_{u:\xi(u)=0} sign(det(\xi'(u)))$$

where

$$det\xi'(u) = \frac{\partial \xi_x}{\partial x}(u)\frac{\partial \xi_y}{\partial y}(u) - \frac{\partial \xi_y}{\partial x}(u)\frac{\partial \xi_x}{\partial y}(u)$$

In the work of Kuznetzov [2], the following formula was proven:

$$\mathbb{E}\gamma(B) = \int_B \mathbb{E}[det(\xi'(u))|\xi(u) = 0]p_{\xi(u)}(0)dydu$$

For that, we need to prove the following:
Lemma 1.1

$$P(\{\exists u \in B : \xi(u) = 0, det\xi'(u) = 0\}) = 0$$

Since $\frac{\partial \xi_x}{\partial x}, \frac{\partial \xi_y}{\partial y}$ are independent, with variance 2, their product is distributed as sum of two chi-square values with 1 degree of freedom.
Then we can prove the following:
Lemma 1.2
$\gamma(\xi, B)$ is stationary, i.e.

$$\gamma(\xi, B) = \gamma(\xi, B + s)$$

in distribution, for all $s \in \mathbb{R}^2$.

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A necessary and sufficient condition for the existence of joint points of images of the trajectories of several independent Brownian motions on Carnot group

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Assume that G_i, $i = 1, \ldots, n$ are Carnot groups in $\mathbb{R}^d$ with the corresponding basis in the Lie algebra of left-invariant vector fields L_{ij}, $j = 1, \ldots, d$, and with homogeneous degrees $p_i : \{1, \ldots, d\} \to \mathbb{N}$, $i = 1, \ldots, n$. Suppose that $X_i(t)$, $t \geqslant 0$, $i = 1, \ldots, n$ are independent Brownian motions on Carnot groups G_i. For definitions of these objects see [1, 2].

Suppose $f_i : \mathbb{R}^d \to \mathbb{R}^m$, $m \leqslant d$ are infinitely differentiable functions such that $rank\, f_i'(x) = m$ for all $i = 1, \ldots, n$, $x \in \mathbb{R}^d$. For $x = (x_1, x_2, \ldots, x_n) \in G_1 \times \ldots \times G_n$ denote

$$F(x) = (f_1(x_1) - f_2(x_2), \ldots, f_{n-1}(x_{n-1}) - f_n(x_n))$$

Definition 1. We say that $x \in G_1 \times \ldots \times G_n$ is a non-singular point of F if any family that consists of vectors $L_{ij}^{x_i} F(x)$, where $L_{ij}^{x_i}$ is the action of the vector field L_{ij} on the variable x_i, are either linearly independent at $y = x$ or linearly dependent for all y, satisfying $F(y) = F(x)$, in some neighbourhood of x.

For any differentiable function $g : \mathbb{R}^d \to \mathbb{R}^m$ and $x \in \mathbb{R}^d$ denote

$$m_i(g, x) = \min\{\sum_{q=1}^{m} p_i(j_q) | 1 \leqslant j_1 < \ldots < j_m \leqslant d :$$
$$L_{i,j_1} g(x_i), \ldots, L_{i,j_m} g(x_i) \text{ are linearly independent}\}$$

Let $H = \{x \in G_1 \times \ldots \times G_n | F(x) = 0\}$. Denote $M(H, T) = \{(x_1, \ldots, x_n) \in H | \exists (t_1, \ldots, t_n) \in T : X_i(t_i) = x_i, i = 1, \ldots, n\}$. The following Theorem uses the results of [1,2] to produce a necessary and sufficient condition for $M(H \cap V, [0, 1]^n)$ to be non-empty with positive probability, but only for the case when an open set V contains only non-singular points of F.

Theorem 1. *Denote as U_0 the set of all non-singular points of F in H. Suppose that $n \geqslant 2$.*

1. *If $m = 1$ or $m = 2$, $n = 2$ then the set $M(U_0, [0, 1]^n)$ is infinite with positive probability.*
2. *If $m \geqslant 4$ or $m = 3$, $n \geqslant 3$ then the set $M(U_0, (0, +\infty]^n)$ is empty with probability 1.*

3. *Suppose that* $m=2$, $n\geqslant 3$. *If there is a point* $x=(x_1,\ldots,x_n)\in U_0$, *such that*
$$\frac{1}{q_1}+\frac{1}{q_2}+\ldots+\frac{1}{q_n}>n-2;$$
where $q_i=m_i(f_i,x_i)-1$, $i=1,\ldots,n$, *then the set* $M(U_0,[0,1]^n)$ *is infinite with positive probability. Otherwise the set* $M(U_0,(0,+\infty]^n)$ *is empty with probability* 1.

4. *Suppose that* $m=3$, $n=2$. *If there is a point* $x=(x_1,\ldots,x_n)\in U_0$, *such that*
   - *either there exist* i,j,k *such that* $p_1(i)=p_1(j)=p_2(k)=1$ *and*
   $$L_{1,i}f_1(x_1),L_{1,j}f_1(x_1),L_{2,k}f_2(x_2) \text{ are linearly independent}$$
   - *or there exist* i,j,k *such that* $p_1(i)=p_2(j)=p_2(k)=1$ *and*
   $$L_{1,i}f_1(x_1),L_{2,j}f_2(x_2),L_{2,k}f_2(x_2) \text{ are linearly independent}$$

   then the set $M(U_0,[0,1]^n)$ *is infinite with positive probability. Otherwise the set* $M(U_0,(0,+\infty]^n)$ *is empty with probability* 1.

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Some properties and asymptotics related to the generalized intersection local times of multidimensional Brownian motion

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Let $\big(B(t)\big)_{0\leqslant t\leqslant 1}$ be a Brownian motion in $\mathbb{R}^d$. The double self intersection local time at a point $u \in \mathbb{R}^d \setminus \{0\}$ is the local time at u of the random field $X(s,t) = B(t) - B(s)$. We denote it by $\rho(u)$. Formally, it can be written as

$$\rho(u) = \int_{0\leqslant s<t\leqslant 1} \delta_u\big(B(t) - B(s)\big)dsdt.$$

If $d \geqslant 4$ then $\rho(u)$ is a positive generalized Wiener function i.e. an element from some Sobolev space $\mathbb{D}^{2,\gamma}$ constructed over the classical Wiener space, denoted by W_0^d. Consequently, it can be represented by a measure θ_u on the Wiener space. We provide some asymptotics for the generalized self intersection local time $\rho(u)$, when u tends to 0, in terms of the measures θ_u. The main results are related to the measure of the whole space $\theta_u(W_0^d)$, the capacity of the support of θ_u and the quadratic Wassersttein distance between θ_u and the Wiener measure.

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Operator splitting for non-homeomorphic one-dimensional stochastic flows

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Flows $\{X_{s,t}(\cdot) \mid 0 \leq s \leq t\}$ of random transformation of the real line that represent interacting Brownian particle systems with common noise and under the action of an external force are considered. The motion of one separate particle is given via an SDE

$$dX_{s,t}(x) = a(X_{s,t}(x))dt + dw_{s,x}(t), \quad t \geq s, x \in \mathbb{R},$$

where $w_{s,x}, s \geq 0, x \in \mathbb{R}$, are correlated Brownian motions, while the pairwise correlation between particles is assumed to depend on the distance between them via

$$\frac{d}{dt}\langle w_{s,x}, w_{s,y}\rangle(t) = \varphi(X_{s,t}(x) - X_{s,t}(y)), \quad t \geq s, x, y \in \mathbb{R},$$

and particles are forced to merge upon a collision. Under proper compatibility assumptions on the drift a and covariance φ the mappings $X_{s,t}$, $s \leq t$, are piecewise constant. An extreme example of such a system is the Brownian web, in which case particles move independently before a collision happens.

The method of operator splitting (the Trotter-Kato formula) is applied to such flows so that the actions of the semigroups generated by the flow with zero drift and the ordinary ODE (for drift) are separated. Weak convergence of finite-dimensional motions is established and this result is used to derive the convergence of the pushforward measures under the action of the corresponding flows (under some additional assumptions). As another application, the convergence of the associated dual flows in reversed time is obtained. The case of the Brownian web is treated separately, in which case the speed of convergence is established.

The talk is based on [1-3], which is partially the joint work with A.A. Dorogovtsev.

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Strong Feller Regularisation of 1-d Nonlinear Transport by Reflected Ornstein-Uhlenbeck Noise

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We studied regularisation by noise for a transport equation on the 1-dimensional Torus, which is induced from a system of ODEs. Consider the equation with interaction, which is in our case the Lagrangian form of a transport equation or an equation with interaction.

$$\begin{cases} dx(u,t) &= b(x(u,t),\mu_t)\mathrm{d}t \\ x(u,0) &= u \in \mathbb{T} \\ \mu_t &= \mu \circ x^{-1}(\cdot,t) \end{cases}$$

where μ is a probability measure. By identifying the measure-valued process with the inverse cumulative distribution functions f_t by $\mu_t = \lambda \circ f_t^{-1}(\cdot)$ where λ is the Lebesgue measure. Then

$$\begin{aligned} df_t(u) &= b(f_t(u), \lambda \circ f_t^{-1}(\cdot))\mathrm{d}t \\ f_0 &= f. \end{aligned} \tag{2}$$

After formally differentiating the equation (2) we regularise the equation by adding space-time white noise and the Laplace operator to end up with:

$$\begin{aligned} dg_t(u) &= \Delta g_t(u)\mathrm{d}t + b'(f_t(u), \lambda \circ f_t(\cdot)^{-1})g_t(u)\mathrm{d}t + \mathrm{d}W(u,t) + \eta \\ g_0 &= f'. \end{aligned}$$

with a reflection term η forcing the solution to stay positive and thus guaranteeing that the solution stays a derivative of an inverse cumulative distribution function. In this talk we will show strong Feller regularisation for the process $(g_t)_{t\geq 0}$.