ABSTRACTS

of the International Conference

"Functional Methods in Operator Theory, Statistics and Stochastic Analysis"

dedicated to 90th anniversary of

Professor A. Ya. Dorogovtsev

November 17-21, 2025, Kyiv, Ukraine

Contents

PLENARY TALKS

STOCHASTIC ANALYSIS

STATISTICS OF STOCHASTIC PROCESSES

INFINITE-DIMENSIONAL DETERMINISTIC AND STOCHASTIC SYSTEM

PLENARY TALKS

Stochastic semigroups, white noise processes and DNA folding

Andrey A. Dorogovtsev[1], **Iryna I. Nishchenko**[2]
[1]Institute of Mathematics of the National Academy of Sciences of Ukraine, Kyiv, Ukraine
[2]National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute", Kyiv, Ukraine
andrey.dorogovtsev@gmail.com, irynan@gmail.com

In the talk we introduce the notion of the white noise process in a separable real Hilbert space and present the construction of the family of coalescing white noise processes. This construction is based on the stochastic semigroup of linear operators. Such semigroups naturally arise as limits of families of coalescing random walks on integer lattice or Ehrenfests cube. Limit theorems for stochastic semigroups are discussed. Using the coalescing white noise processes we build a model of moving random knot whose topological type changes with time, but finally this knot almost "freezes" and moves like a solid body (a possible model for DNA folding).

Periodogram Estimates of Trigonometric Regression Models Parameters: Development of Professor A. Ya. Dorogovtsev Results

A. V. Ivanov
Igor Sikorsky Kyiv Polytechnic Institute, Kyiv, Ukraine
alexntuu@gmail.com

Periodogram estimates (PE) of trigonometric models parameters were first studied by A. Shuster (1898, 1906). He introduced the term "periodogram" into common usage. After him, several generations of mathematicians studied PE under various assumptions about the signals and random noises masking these signals.

In the 1st part of the presentation, we discuss fruitful A. Ya. Dorogovtsev's results [1] on asymptotic PE behavior of angular frequency and amplitude of harmonic oscillating observed in the background of stationary random noise satisfying strong mixing condition (SMC).

The next parts of the lecture devoted to various generalizations of the article [1] theorems.

In the 2nd part we consider the field generalizations collected in the P. S. Knopov's book [2]. There in the capacity of useful signals the functions of several variable are chosen that are periodic in each variable with period of 2π, and each variable multiplied by unknown parameter (frequency). The noise is a homogeneous random field satisfying SMC. Besides PE of these parameters the periodogram-like estimates are considered also. Under additional conditions on Fourier coefficients of this periodic functions strong consistency and coordinate-wise asymptotic normality were obtained for both estimates.

Part 3 is about PE in trigonometric model provided that the random noise is a local functional of stationary Gaussian process with spectral density having singularities including at zero, i.e., it is strongly dependent. It is proven that PE of model parameters is weakly consistent and asymptotically normal. The diagram technique is used to obtain these results. The necessary mathematical apparatus for studying the properties of PE in this model is described in [3].

In the part 4 we consider the PE of multivariable trigonometric model parameters with noise that is a homogeneous and isotropic Gaussian field satisfying the strong dependency condition. The results obtained for this model are contained in the work [4].

1. Grechka G. P., Dorogovtsev A. Ya. On periodogram estimate asymptotic properties of the frequency and amplitude of harmonic oscillation, Computational and Applied Math, 1976, 28, 18-31.
2. Knopov P.S. Optimal estimates of stochastic systems parameters. – Kyiv, Naukova Dumka, 1981, P.152.
3. Ivanov A.V., Leonenko N.N., Ruiz-Medina M.D., Zhurakovsky B.M. Estimation of harmonic component in regression with cyclically dependent errors. Statistics: A Journal of Theor. and Appl. Statistics, 2015, 49, 1, 156-186.
4. Ivanov A.V., Savych I.M., Dykyi O.V. Asymptotic properties of the periodogram estimates of the multivariate symmetric textured surface parameters. Probab. Theory and Math. Statist., 2025, issue 113 (To appear).

Asymptotic properties of the empirical means method in stochastic optimization

P. S. Knopov[1], O. S. Samosonok[2]

[1] V.M. Glushkov Institute of Cybernetics of the National Academy of Sciences of Ukraine, Kyiv, Ukraine

[2]V.M. Glushkov Institute of Cybernetics of the National Academy of Sciences of Ukraine, Kyiv, Ukraine

knopov1@yahoo.com, samosyonok@gmail.com

The necessity to make decisions under risk and uncertainty is one of the current issues in modern optimization theory. The key characteristics of this class of problems include the lack of complete information about the objective and constraint functions, their derivatives, and their non-smooth nature. The most challenging problems are those in which precise information about the values of the functions or constraints themselves is missing; these can be formalized within the general theory of stochastic programming. Numerous methods and techniques exist for solving this class of problems. However, when solving stochastic optimization and identification problems, it is not always possible to find the exact extremum of a random function expectation. The empirical means method, involving an existing criterion function approximating with its empirical estimate, for which it is possible to solve the corresponding optimization problem, is an approach to addressing this issue. Naturally, conditions for approximate estimates convergence obtained by this method depend significantly on a criterion function, random observations probabilistic properties, a metric of spaces for which convergence is being studied, a priori constraints on unknown parameters, etc. In the statistical decision theory terminology, these issues are closely related to the asymptotic properties of unknown parameter estimates, namely, consistency, asymptotic distribution, and the rate of convergence of estimates.

It should be noted that a fairly large number of publications have been devoted to the empirical means method. One of the most important approached is based on so-called epi-convergence, which is based on convergence conditions using the epi-distance concept. In this paper, we consider other approaches to proving convergence assertions, using the results of [1]. These approaches are widely used in modern asymptotic estimation theory and prove useful for solving the optimization problems under consideration. This paper focuses on certain problems of the empirical means method limit behavior for models with dependent observations. Assertions regarding the method convergence for discrete and continuous time and the limit distribution of estimates are proved. Cases in which the observations represent a homogeneous random field satisfying the strong mixing condition are considered. Finally, results are presented on the problem of large deviations for various models of the empirical means method, allowing us to estimate the rate of convergence of the method and investigate the asymptotic properties of estimates for unknown parameters in various regression models.

1. Dorogovtsev A. Ya. The Theory of Estimation of Random Processes Parameters.— Kyiv: Vyshcha Shkola, 1982.
2. Knopov P. S. Kasitskaya E. J. Empirical Estimates in Stochastic Optimization and Identification.— Dordrecht: Kuwer, 2005.
3. Knopov P. S., Kasitskaya E. J. Large deviations of empirical estimates in stochastic programming with non-stationary observations. Cybern. Syst. Anal., 2010, 5, 40вЂ“45.
4. Knopov P. S., Kasitskaya E. J. On large deviations of empirical estimates in stochastic programming. Cybern. Syst. Anal., 2004, 4, 52-61.
5. Knopov P. S., Kasitskaya E. J. Large deviations for the method of empirical means in stochastic optimization problems with continuous time observations. Optimization Methods and Applications. In Honor of the 80-th Birthday of Ivan V. Sergienko. S. Butenko, P.M. Pardalos, and V. Shylo, Eds. Cybern. Syst. Anal., 2004, 4, 52-61.

   Knopov P., Kasitskaya E. Large deviations for the method of empirical means in stochastic optimization problems with continuous time observations. Optimization Methods and Applications. In Honor of the 80-th Birthday of Ivan V. Sergienko. S. Butenko, P.M. Pardalos, and V. Shylo, Eds. New York: Springer, 2017. P. 263-275.

From prime numbers to damped waves, and beyond

Yuri Tomilov
Institute of Mathematics, Polish Academy of Sciences, Warsaw, Poland
ytomilov@impan.pl

I will present recent results establishing Tauberian theorems for L^p functions with optimal L^p convergence rates, and will show how they yield new energy-decay properties for damped wave equations. Related developments will also be discussed. Joint work with C. Batty (Oxford), A. Borichev (Marseille), and R. Chill (Dresden).

Regularity of Volterra Quadratic Stochastic Operators Generating Associative Genetic Algebras

N. N. Ganikhodjaev[1], G. Dusmurodova[2]
[1]V.I. Romanovsky Institute of Mathematics,Tashkent, Uzbekistan
[2]University of Geological Sciences,Tashkent,Uzbekistan
nasirgani@yandex.com, dusmurodova77@gmail.com

Let $S^{m-1} = \{\mathbf{x} = (x_1, x_2, \cdots, x_m) \in R^m : \text{ for any } i \ x_i \geq 0, \text{ and } \sum_{i=1}^m x_i = 1\}$ be the $(m-1)$–dimensional simplex.

Definition 1. A mapping $V : S^{m-1} \to S^{m-1}$

$$(V\mathbf{x})_k = \sum_{i,j=1}^m p_{ij,k} x_i x_j$$

with

$$a) p_{ij,k} \geq 0, \quad b) p_{ij,k} = p_{ji,k} \text{ for all } i, j, k; \quad c) \sum_{k=1}^m p_{ij,k} = 1$$

is called quadratic stochastic operator.

In [1], $m-$dimensional algebra with genetic realization is defined as algebra A over the real numbers R which has a basis $\{a_1, a_2, \cdots, a_m\}$ and a multiplication table

$$a_i a_j = \sum_{k=1}^m p_{ij,k} a_k.$$

Since a genetic algebra is defined by a quadratic stochastic operator, it is natural to study whether there is a relationship between the ergodic properties of this operator and the algebraic properties of the corresponding genetic algebra.

The algebras that arise in genetics are generally commutative but non-associative. In this paper we will discuss the following conjecture: from associativity of genetic algebra generated by operator V follows its regularity.

Definition 2. The quadratic stochastic operator V is called Volterra, if $p_{ij,k} = 0$ for any $k \notin \{i, j\}$.

The biological treatment of such operators is rather clear: the offspring repeats one of its parents. It is evident that operator V be a Volterra if and only if

$$(V\mathbf{x})_k = x_k(1 + \sum_{i=1}^m a_{ki} x_i),$$

where $A = (a_{ij})_1^m$ is a skew-symmetric matrix with $a_{ki} = 2p_{ik,k} - 1$ for $i \neq k$, $a_{ii} = 0$ and $|a_{ij}| \leq 1$. Here $i, j \in \{1, 2, \cdots, m\}$.

Definition 3. The quadratic stochastic operator V is called an extremal Volterra, if $a_{ij} = -1$ or 1 for any $i \neq j$.

Definition 4. A so V is called regular if for any initial point $\mathbf{x} \in S^{m-1}$ the limit

$$\lim_{n\to\infty} V^n(\mathbf{x})$$

exists.

It is well-known that m-dimensional algebra A with a genetic realization is associative iff for $i, j, k, s = 1, \cdots, m$

$$\sum_{r=1}^{m} p_{ij,r}p_{rk,s} = \sum_{r=1}^{m} p_{ir,s}p_{jk,r}. \tag{1}$$

Thus to verify the associativity of algebra A we have to show the validity of m^4 equalities (1). Note that some of these equalities are identities and others are equations.

Theorem 1. *i) Any Volterra operator on S^{m-1} generating associative genetic algebra is the extremal operator;*
ii) From all $2^{m(m-1)/2}$ extremal Volterra operator only $m!$ such operators generate associative genetic algebra;
iii) All extremal Volterra generating associative algebra are regular transformation and any extremal regular Volterra operator generate associative algebra.

1. M. L. Reed, Algebraic Structure of Genetic Inheritance. Bulletin of the American Mathematical Society, 1997, **34**, 2,107-130.

Neighbour-Count Thinning of Point Processes

Kateryna Hlyniana[1,2]

[1]Institute of Mathematics, National academy of science of Ukraine, Kyiv

[2]School of Mathematics, Jilin University, China

glinkate@gmail.com

In this talk we consider a thinning of a point process that depends on its local configuration. Let X be a point process on $\mathbb{R}^d$ and fix a range $r > 0$. For $x \in X$, let $n_r(x;X) := \#\big((X \setminus \{x\}) \cap B(x,r)\big)$ be the number of neighbours within distance r. For a measurable function $p : \mathbb{N}_0 \to [0,1]$, the *$r$-local neighbour-count thinning* T_r acts independently at each $x \in X$ by retaining x with probability $p(n_r(x;X))$. We write $Y = T_r(X)$ for the retained configuration.

The rule T_r is translation invariant and provides a flexible mechanism to encode attraction or inhibition based on the local crowding level. When X is a Poisson input, one can compute first- and second-order characteristics of Y explicitly in terms of p and the geometric overlap of r balls, which leads to tractable small r (contact-scale) expansions for the pair correlation and quantitative Stein-type Poisson approximations.

Let X be a Poisson point process with intensity λ on $\mathbb{R}^d$ and let $Y := T_r(X)$. We use the radial pair-correlation $g_Y^\circ : [0,\infty) \to [0,\infty)$, defined by

$$g_Y^\circ(r) := \frac{\rho_Y^{(2)}(x,y)}{\rho_Y(x)\rho_Y(y)}$$

for any (x,y) with $\|x-y\| = r$. Define the normalized overlap coefficient

$$\omega_d(t) := \frac{|B(0,1) \cap B(te_1,1)|}{v_d} \in [0,1], \qquad \omega_d(t) = 0 \text{ for } t \geq 2.$$

Theorem 1 (First-order contact-scale expansion of g). *Assume $p(0), p(1) > 0$. For $t = \|h\|/r \in (0,2]$ put $I := \mathbf{1}\{t \le 1\}$ and $\mu := \lambda v_d r^d$, where $v_d := |B(0,1)|$. Then,*

(i) If $t \in (0,1]$, as $r \to 0$,

$$g_Y^\circ(tr) = \frac{p(1)^2}{p(0)^2}\left\{1 + \mu\Big[\omega_d(t)\Big(1 - \frac{p(2)}{p(1)}\Big)^2 + 2\Big(\frac{p(2)}{p(1)} - \frac{p(1)}{p(0)}\Big)\Big]\right\} + O(r^{2d}).$$

(ii) If $t \in (1,2]$, as $r \to 0$,

$$g_Y^\circ(tr) = 1 + \mu\,\omega_d(t)\Big(1 - \frac{p(1)}{p(0)}\Big)^2 + O(r^{2d}).$$

To estimate how $T_r(X)$ deviates from a Poisson point process with the same intensity, we define a distance between distributions of point processes [1,2]. Let (W, d_0) be a bounded metric space with $d_0 \le 1$. For $\xi = \sum_{i=1}^n \delta_{x_i},\ \eta = \sum_{j=1}^m \delta_{y_j}$ set

$$d_1(\xi,\eta) = \begin{cases} \min\limits_{\pi \in S_n} \dfrac{1}{n}\displaystyle\sum_{i=1}^n d_0(x_i, y_{\pi(i)}), & n = m \ge 1, \\ 1, & n \ne m, \\ 0, & n = m = 0. \end{cases}$$

For laws P, Q on space of simple point processes with support in W, $\mathbf{N}(W)$ define

$$d_2(P,Q) := \sup\Big\{\big|\mathbb{E}_P f - \mathbb{E}_Q f\big|, \text{ for } f : \mathbf{N}(W) \to [0,1],\ |f(\xi) - f(\eta)| \le d_1(\xi,\eta)\ \forall\, \xi,\eta \in \mathbf{N}(W)\Big\}.$$

Theorem 2 (Stein–Poisson approximation). *Let $X \sim \mathrm{PPP}(\lambda)$ on $\mathbb{R}^d$, let $Y := T_r(X)$ be the neighbour-count dependent thinning with retention $p : \mathbb{N}_0 \to [0,1]$, and fix a bounded Borel window $W \subset \mathbb{R}^d$. Write $\lambda' = \lambda\, m_p$ with $m_p = \mathbb{E}[p(N_r)]$, $N_r \sim \mathrm{Poisson}(\lambda v_d r^d)$, and let $\Pi_W \sim \mathrm{PPP}(\lambda' \mathbf{1}_W\, dx)$. Then*

$$d_2\left(\mathcal{L}(Y_W), \mathcal{L}(\Pi_W)\right) \ \le\ \lambda'^2\, |W| \left[C_d\, r^d \int_0^2 t^{d-1}\, |g_Y^\circ(tr) - 1|\, dt \ +\ v_d\, (2r)^d\right],$$

where $C_d \in (0,\infty)$ depends only on d.

References

1. Schuhmacher D. Stein's method and Poisson process approximation for a class of Wasserstein metrics. *Bernoulli*, 2009, 15(1), 550-565.

2. Barbour A.,D., Brown T.,C. Stein's method and point process approximation. *Stochastic Processes and their Applications*, 1992, 43(1), 9-31.

Topological types of a moving random link

Yurii S. Miniailyk
Odessa I. I. Mechnikov National University
yuriiminiailyk@gmail.com

Let us consider the construction of a moving random link. Suppose that $\vec{W}: \mathbb{R}^2 \times \mathbb{R} \to \mathbb{R}^3$ is a Wiener sheet and $h \in L^2(\mathbb{R}^2)$ is such that

$$(h * h)(\vec{u}) = e^{-\frac{1}{2}||\vec{u}||},\ h(-\vec{u}) = h(\vec{u}).$$

Then consider a following SDE

$$d\vec{\xi}^t(\vec{u}) = -\vec{\xi}_t(\vec{u})dt + \sqrt{2}\int_{\mathbb{R}^2} h(\vec{u}-\vec{p})\vec{W}(d\vec{p}, dt).$$

This SDE defines Gaussian random fields $\vec{\xi}^t: \mathbb{R}^2 \to \mathbb{R}^3$ which are defined by the next formula.

$$\vec{\xi}^t(\vec{u}) = \int_{\mathbb{R}^2}\int_{-\infty}^{t} \sqrt{2}e^{-(t-s)}h(\vec{u}-\vec{p})\vec{W}(d\vec{p}, ds)$$

Standard calculations give that for any nonnegative t $\vec{\xi}_t$ is a centered Gaussian random field with independent identically distributed coordinates with covariance

$$\mathbb{E}\xi_1^t(\vec{v})\xi_1^t(\vec{u}) = e^{-\frac{1}{2}||\vec{u}-\vec{v}||^2} = G(\vec{u}-\vec{v}).$$

Since the covariance is infinitely differentiable, due to Gaussianity, $\vec{\xi}^t$ has an infinitely differentiable on $\mathbb{R}^2$ modification [3].

Suppose that $\theta_i = (x_i + \cos t, y_i + \sin t) : [0, 2\pi] \to \mathbb{R}^2,\ i = 1, 2$ are two non intersecting unit circles on the plane. Then consider $\gamma_i^t = \vec{\xi}^t(\theta_i),\ i = 1, 2$. In [2] it was proved that with probability 1 $\gamma^t = \gamma_1^t \cup \gamma_2^t$ is a smooth random link. We will use the following definition for two links to have the same topological type.

Definition 1. *Smooth links Γ and Γ' are equivalent if exists $F \in C^1(\mathbb{R}^3 \times [0, 1], \mathbb{R}^3)$ such that three conditions hold.*
1) $F(\vec{x}, 0) = \vec{x},\ \vec{x} \in \mathbb{R}^3$.
2) For any $s \in [0, 1] : F(\cdot, s)$ is a diffeomorphism of $\mathbb{R}^3$.
3) $F(\Gamma, 1) = \Gamma'$.
F is called smooth ambient isotopy.

In paper [1] A. A. Dorogovtsev proved that for any nonnegative fixed t random knots γ_1^t, γ_2^t attain any topological type with positive probability. Making small changes to the proof we can get similar result for random links.

Theorem 1. *For any fixed nonnegative t with probability 1 γ^t is a regular smooth link. Let Γ be a regular smooth link in $\mathbb{R}^3$ with two components. Then set of $\omega \in \Omega$ such that γ and Γ are equivalent is a random event of positive probability.*

Moreover, it occurs that during the motion our moving random link attains any topological type infinitely often. To be precise, the next theorem holds.

Theorem 2. *Let Γ be a regular smooth link with two components. Suppose that $\{t_n\}$ is an increasing sequence in $\mathbb{R}$ such that $\lim_{n\to\infty}(t_{n+1} - t_n) = \infty$. Then*

$$P\{\gamma^{t_n} \sim \Gamma\ i.o.\} = 1$$

References

[1] Dorogovtsev, Andrey A. (2023) "On a Stationary Random Knot", Journal of Stochastic Analysis: Vol. 4: No. 3, Article 4.

[2] Y. Miniailyk "Geometric invariants of random link", Theory of Stochastic Processes, Vol. 29 (45), no. 1, 2025, pp. 65-73.

[3] Adler, Robert J., and Jonathan E. Taylor. Random Fields and Geometry. Springer, 2009.

The Geometric Property of Random Curves and Stochastic Flows

Qingsong Wang[1], A. A. Dorogovtsev[2]

[1]School of Mathematics, Jilin University, Changchun, 130012, P. R. China.

[2]Institute of Mathematics, National Academy of Sciences of Ukraine, Ukraine.

qswang21@mails.jlu.edu.cn, andrey.dorogovtsev@gmail.com

The aim of the talk is going to describe simple geometric structure arising in Brownian trajectory and study its evolution with the time. The talk is divided into three parts: The first part considered the hitting probability of Brownian motion and conditional expectation of self-intersection local time, presented their asymptotic. We studied the self-intersection local time of Brownian motion and derived its asymptotic results under conditional expectations. Additionally, we defined the concept of a polygonal line being "almost inscribed" in a Brownian path, and analyzed the conditional distribution of the path and its asymptotic results when the polygonal line satisfies this property. The second part, we investigated the hitting time and hitting probability of a specific class of Gaussian processes (referred to as integrators). By expressing their hitting probability as a functional of Brownian motion and utilizing the Itô-Wiener expansion results for Brownian functionals, we represented the hitting probability of the integrator as a multiple integral series. The third part, we focused on stochastic differential equations with interactions. We first provided the definition of such equations, presented examples of equations with unique solutions in simple cases, and derived the explicit forms of these solutions. Then, we introduced two examples of stochastic differential equations with interactions in the context of physics. Next, we incorporated visitation measures into the framework of stochastic differential equations with interaction. We further studied interacting stochastic flows, constructed systems with multiple attract points, analyzed the visitation measures of these flows, and presented several asymptotic results.

Acknowledgements

1. A. A. Dorogovtsev, Stochastic integration and a class of Gaussian random processes. Ukrainian Math. J. 1998, 50, no.4, 550–561.
2. A. A. Dorogovtsev and N. Salhi, Refinements of asymptotics at zero of Brownian self-intersection local times. Infin. Dimens. Anal. Quantum Probab. Relat. Top. 2024, 27.
3. A. A. Dorogovtsev, Qingsong Wang, Hitting time for one class of Gaussian processes. 2024.
4. K. Itô and H. P. McKean, Diffusion Process and Their Sample Paths. Springer-Verlag. 1974.
5. A. N. Shiryaev, Probability. Springer., 2008.
6. A. A. Dorogovtsev, O. L. Izyumtseva and N. Salhi, Clark representation for local times of self-intersection of Gaussian integrators. Ukr. Mat. Zh. 1587-1614, 70, 2018.
7. A. A. Dorogovtsev, Measure-valued processes and stochastic flows. De Gruyter 2023.

Lévy measures on Banach spaces

M. Riedle[1]
[1]Department of Mathematics, King's College London, Strand, London WC2R 2LS, UK
markus.riedle@kcl.ac.uk

A fundamental challenge in the study of Lévy processes in Banach spaces is the absence of an explicit characterisation of Lévy measures. Historically, a complete description has only been available in sequence spaces, while in more general Banach spaces only partial results were known, such as sufficient or necessary conditions under additional geometric assumptions (e.g., type and cotype). Moreover, the classical integrability condition used in the definition of Lévy measures fails to capture their correct structure in many infinite-dimensional settings.

This talk provides an introduction to the problem of characterising Lévy measures in Banach spaces and explains how to define Lévy measures without relying on the classical integrability condition. We illustrate the difficulties through classical examples that demonstrate the breakdown of standard criteria.

Our main results give explicit characterisations in two important settings. For L^p-spaces, Lévy measures are characterised by a natural integrability condition that generalises the known description for sequence spaces; this yields the first explicit characterisation of Lévy measures in non-Hilbert infinite-dimensional Banach spaces. For UMD Banach spaces, Lévy measures are characterised by the finiteness of the expected value of a suitable random γ-radonifying norm. While this formulation is more abstract, it reduces to the concrete integrability condition in the L^p-case, thereby providing a unified framework.

This talk is based on joint work with Jan van Neerven (TU Delft).

1. van Neerven, J., Riedle, M. Lévy measures on Banach spaces. Annales de l'Institut Henri Poincaré, Probabilités et Statistiques, 2026 (forthcoming).

Self-intersections for an image of the trajectory of Brownian motion on Carnot group

O.V. Rudenko
[1]Institute of mathematics of NAS of UKraine, Kyiv, Ukraine
arooden@gmail.com

Assume that $d \geqslant 2$ and G is a d-dimensional Carnot group with the corresponding basis in the Lie algebra of left-invariant vector fields L_j, $j = 1, \ldots, d$, and with homogeneous degrees $p : \{1, \ldots, d\} \to \mathbb{N}$, $i = 1, \ldots, n$. Suppose that $X(t)$, $t \geqslant 0$ is a Brownian motion on Carnot groups G. For definitions of these objects see [1,2].

Suppose $f : G \to \mathbb{R}^m$, $m \leqslant d$ is infinitely differentiable function such that $rank\, f'(x) = m$ for all $x \in G$. For $x = (x_1, x_2, \ldots, x_n) \in G^n$ denote

$$F(x) = (f(x_1) - f(x_2), \ldots, f(x_{n-1}) - f(x_n))$$

Definition 1. We say that $x \in G^n$ is a non-singular point of F if for any selection of $k_i = 1, 2, \ldots$ and $t_i : \{1, \ldots, k_i\} \to \{1, \ldots, d\}$ for $i = 1, \ldots, n$ the following condition is either holds at $y = x$ or it does not hold for all $y = (y_1, \ldots, y_n) \in G^n$ with $F(y) = F(x)$ in some neighbourhood of x: the vectors $L^{y_1}_{t_1(1)}F(y), \ldots, L^{y_1}_{t_1(k_1)}F(y), L^{y_2}_{t_2(1)}F(y), \ldots, L^{y_n}_{t_n(k_n)}F(y)$ are linearly independent, where $L^{y_k}_i F(y)$ is an action of the vector field L_i on the function F as a differential operator on the variable y_k.

For any $x \in G$ denote

$$m(x) = \min\{\sum_{q=1}^{m} p(j_q) | 1 \leqslant j_1 < \ldots < j_m \leqslant d :$$
$$L_{j_1} f(x), \ldots, L_{j_m} f(x) \text{ are linearly independent}\}$$

Let $H = \{x \in G^n | F(x) = 0\}$. Denote $M(H, T) = \{(x_1, \ldots, x_n) \in H | \exists (t_1, \ldots, t_n) \in T, t_1 < t_2 < \ldots < t_n : X(t_i) = x_i, i = 1, \ldots, n\}$.

Theorem 1. *Suppose that U_0 the set of all non-singular points of F in H and $V \subset G^n$ is an open set.*

1. *If $m = 1$, $n \geqslant 2$ or $m = 2$, $n = 2$ then the set $M(U_0, [0, 1]^n)$ is infinite with positive probability.*
2. *If $m \geqslant 4$ or $m = 3$, $n \geqslant 3$ then the set $M(U_0, (0, +\infty]^n)$ is empty with probability 1.*
3. *Suppose that $m = 2$, $n \geqslant 3$. If there is a point $x = (x_1, \ldots, x_n) \in U_0 \cap V$, such that*

$$\frac{1}{q_1} + \frac{1}{q_2} + \ldots + \frac{1}{q_n} > n - 2;$$

   where $q_i = m(x_i) - 1$, $i = 1, \ldots, n$, then the set $M(U_0 \cap V, [0, 1]^n)$ is infinite with positive probability. Otherwise the set $M(U_0 \cap V, (0, +\infty]^n)$ is empty with probability 1.

4. *Suppose that $m = 3$, $n = 2$. If there is a point $x = (x_1, x_2) \in U_0 \cap V$, such that*
   - *either there exist i, j, k such that $p(i) = p(j) = p(k) = 1$ and*

     $$L_i f(x_1), L_j f(x_1), L_k f(x_2) \text{ are linearly independent}$$

   - *or there exist i, j, k such that $p(i) = p(j) = p(k) = 1$ and*

     $$L_i f(x_1), L_j f(x_2), L_k f(x_2) \text{ are linearly independent}$$

   then the set $M(U_0 \cap V, [0, 1]^n)$ is infinite with positive probability. Otherwise the set $M(U_0 \cap V, (0, +\infty]^n)$ is empty with probability 1.

1. Rudenko A. Intersection local times in L2 for Markov processes, Theory of Stoch. Proc., 2019, 24(40), no. 1, 64–95.
2. Rudenko Oleksii, The set of intersections of several independent Brownian motions on Carnot group, Stochastics, 2024, 96, no. 7, 1928–1947.

STATISTICS OF STOCHASTIC PROCESSES

Weighting of a singular values vector for the random projection method

O. O. Desiateryk[1], **E. G. Revunova**[2]
[1]Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
[12]Institute of Information Technologies and Systems, Kyiv, Ukraine
desiateryk@knu.ua, egrevunova@gmail.com

We consider the regularization of the inverse problem based on random projection. In many practical applications, signal transformation is described by a linear model of the form $y = Ax + \varepsilon$, where the matrix $A \in \mathbb{R}^{N \times N}$ and the vector $y \in \mathbb{R}^N$ and $y = y_0 + \varepsilon$, $y_0 = Ax$ are known. The components of the noise vector $\varepsilon \in \mathbb{R}^N$ are realizations of independent Gaussian random variables with zero mean and variance σ^2. The signal vector $x \in \mathbb{R}^N$ has to be estimated.

In the case when y contains noise and the series of singular numbers of the matrix A smoothly drops to zero (with A having a high conditionality number), the problem of estimating x is called the discrete ill-posed problem (DIP) [1]. For DIP, the solution (estimate of vector x) obtained on the basis of a pseudo-inversion as $x^* = A^+ y$, where A^+ is a pseudoinverse is unstable and inaccurate.

To obtain a stable solution (estimation x^*), such methods as truncated singular value decomposition, truncated QR decomposition, and the method based on random projection can be used.

To obtain solution based on random projection, both sides of the original equation are multiplied by the matrix $R_k \in \mathbb{R}^{k \times N}$ resulting in the equation

$$R_k A x = R_k y,$$

where $R_k A \in \mathbb{R}^{k \times N}$, $R_k y \in \mathbb{R}^k$. The vector of the recovered signal is obtained as

$$x_k^* = (R_k A)^+ R_k y.$$

As a random matrix R_k we can use: the matrix $G_k \in \mathbb{R}^{k \times N}$ whose elements are realizations of a random variable with a Gaussian distribution, zero mean and unit variance; the matrix $Q_k \in \mathbb{R}^{k \times N}$ obtained by QR decomposition of GA matrix; the matrix $\Omega_k \in \mathbb{R}^{k \times N}$ obtained by SVD decomposition of G matrix ($G = \Omega \Sigma \Psi^T$).

Experimental studies have shown that there is an optimal number k ($k < N$) of the R rows, which minimizes the error

$$e_x = \|x - x_k^*\|^2$$

of the true vector recovering.

The accuracy of the DIP solution by the method of random projection depends on two independent random variables. The first one is the additive noise in the output vector (whose distribution is assumed to be Gaussian, and the variance is generally unknown) and the second one is the random variable that forms the random matrix (Gaussian distribution with the unit variance, in the studied case). Changing the number of rows k of the random matrix leads to a change in the accuracy of the DIP solution. In the absence of noise in the output vector, an increase in the number of rows of a random matrix leads to a decrease in the solution error. Noise in the output vector leads to the appearance of an error component, the value of which increases with increasing number of rows of a random matrix. Therefore, the dependence of the error of the DIP solution on the number of rows of the random matrix has a minimum at $k < N$ (at certain noise levels).

Experimental studies showed that averaging over random matrices leads to a smoothing of the e_x dependence and a decrease in the number of local minima. Analytic averaging over random matrices can lead to simpler expressions for e_x, facilitate further analytical research and improve the accuracy of the method of DIP solving based on random projection.

The next formula for the expectation E_R (by matrices R) is known

$$E_R\{R_k^T(R_k Z R_k^T)^{-1}R_k\} = diag\,(\lambda_1,\ldots,\lambda_m,\mu,\ldots,\mu) = D_k(Z_m)$$

for a matrix $Z \in \mathbb{R}^{N\times N}$ which is a diagonal matrix of the eigenvalues of the symmetric positive semidefinite matrix $B \in \mathbb{R}^{N\times N}$, $Z_m \in \mathbb{R}^{m\times N}$, and $D_k(Z_m) \in \mathbb{R}^{N\times N}$ is diagonal, where $\lambda_i = \frac{\mu}{1+\mu s_i^2}$, $\mu = const.$

Proposition 1. *Let the e_x be the error of DIP solution. Then the averaging of the error e_x over random matrices is obtained as*

$$E_R\{e_x\} = xx^T - x^T V S^2 D_k V^T x + \sigma^2 trace\,(U D_k U^T),$$

where U, S from $A = USV^T$.

This averaging let us propose the method of deterministic random projection (DRP) for the search of DIP solution with lower error.

Proposition 2. *The estimation x^*_{DRP} of the vector x by DRP is obtained as*

$$x^*_{DRP} = A^T U D_k U^T y. \tag{1}$$

The formula (1) for the estimation x^*_{DRP} can be presented in the next form with weighed singular values vector

$$x^*_{DRP} = V S D_k U^T y = V\,diag(s_i^2 d_{ki}\frac{1}{s_i})U^T y.$$

Here as weights we consider $w_{DRP} = s_i^2 d_{ki}$.

1. Revunova E. G. Model selection criteria for a linear model to solve discrete ill-posed problems on the basis of singular decomposition and random projection. Cybernetics and Systems Analysis, 2016, Vol. 52, N 4, P. 647-664.

Estimation problem for periodically correlated stochastic processes with missing observations

I. I. Golichenko[1]

[1]Igor Sikorsky Kyiv Polytechnic Institute, Kyiv, Ukraine

idubovetska@gmail.com

The problem of mean-square optimal linear estimation of the functional

$$A_s\zeta = \sum_{l=0}^{s-1} \int_{M_l}^{M_l+N_{l+1}} a(t)\zeta(t)dt$$

which depends on the unknown values of a periodically correlated stochastic process $\zeta(t)$, [1], is considered. The estimation is based on observations of the process $\zeta(t) + \theta(t)$ at points $t \in \mathbb{R} \setminus S$, $S = \bigcup_{l=0}^{s-1} [M_l, M_l + N_{l+1}]$, $M_l = \sum_{k=0}^{l} (N_k + K_k)$, $N_0 = K_0 = 0$, where $\theta(t)$ is uncorrelated with $\zeta(t)$ periodically correlated stochastic process.

The estimation problem is studied under assumption that the number of missed observations at each of the intervals is a multiple of the period T: $K_1 = T \cdot K_1^T, K_2 = T \cdot K_2^T, \ldots, K_{s-1} = T \cdot K_{s-1}^T$, and the number of observations at each of the intervals is a multiple of T: $N_1 = T \cdot N_1^T, N_2 = T \cdot N_2^T, \ldots, N_s = T \cdot N_s^T$.

Denoting by

$$a(u + jT) = a_j(u),\ \zeta(u + jT) = \zeta_j(u),\ j \in \widetilde{S},\ u \in [0, T),$$

$\widetilde{S} = \bigcup_{l=0}^{s-1} \left\{ M_l^T, ..., M_l^T + N_{l+1}^T - 1 \right\}$, and taking into account the decomposition of generated vector stationary sequence $\{\zeta_j, j \in \mathbb{Z}\}$, [2], the functional $A_s\zeta$ can be written as

$$A_s\zeta = \sum_{l=0}^{s-1} \sum_{j=M_l^T}^{M_l^T+N_{l+1}^T-1} \int_0^T a(u + jT)\zeta(u + jT)du = \sum_{l=0}^{s-1} \sum_{j=M_l^T}^{M_l^T+N_{l+1}^T-1} \vec{a}_j^\top \vec{\zeta}_j,$$

where vectors $\vec{a}_j$ are of the form

$$\vec{a}_j = (a_{kj}, k = 1, 2, \dots)^\top = (a_{1j}, a_{3j}, a_{2j}, \dots, a_{2k+1,j}, a_{2k,j}, \dots)^\top,\ j \in \tilde{S},$$

$a_{kj} = \frac{1}{\sqrt{T}} \int_0^T a_j(v) e^{-2\pi i \{(-1)^k [\frac{k}{2}]\} v/T} dv$, vector sequence $\vec{\zeta}_j = (\zeta_{kj}, k = 1, 2, \dots)^\top$, $j \in \widetilde{S}$, is generated vector stationary sequence.

In the case of spectral certainty when the spectral density matrices $f^\zeta(\lambda)$ and $f^\theta(\lambda)$ of the generated vector stationary sequences $\{\zeta_j, j \in \mathbb{Z}\}$ and $\{\theta_j, j \in \mathbb{Z}\}$ are exactly known the Hilbert space projection method to estimation of functional $A_s\zeta$ is applied. Formulas for calculating the spectral characteristic and the mean-square error of the optimal estimate of the functional are derived, [3].

In the case of spectral uncertainty when density matrices are not exactly known, but a set $D = D_f \times D_g$ of admissible spectral densities is specified the minimax (robust) approach to estimation of functional $A_s\zeta$ is used. Formulas that determine the least favorable spectral densities and the minimax spectral characteristic of the optimal estimate of the functional are derived, [3]. The estimation problem is investigated in details for sets D_0^-, D_M^- of admissible spectral densities.

1. Gladyshev E. G. Periodically and almost periodically random processes with continuous time parameter. Theory Probab. Appl., 1963. V. 8, P. 173–177.
2. Moklyachuk M. P., Golichenko I. I. Periodically correlated processes estimates. — LAP LAMBERT Academic Publishing, 2016, 308 p.
3. Golichenko I. I., Moklyachuk M. P. Interpolation problem for periodically correlated processes with missing observations. Scientific Bulletin of Uzhhorod University. Series of Mathematics and Informatics, 2025, 47(2). (in print)

Consistency and asymptotic normality of periodogram-like estimates for chirp signal parameters

A. V. Ivanov[1], V. V. Hladun[2]

[1]Igor Sikorsky Kyiv Polytechnic Institute, Kyiv, Ukraine

[2]Igor Sikorsky Kyiv Polytechnic Institute, Kyiv, Ukraine

viktor.hladun00@gmail.com, alexntuu@gmail.com

A time-continuous model of a simple chirp signal observed in the presence of strongly or weakly dependent stationary Gaussian noise is considered. For this statistical model, the strong consistency and asymptotic normality of periodogram-like estimates of the unknown amplitude, angular frequency, and chirp rate are obtained.

Assume that a stochastic process

$$X(t) = A^0 \cos(\phi^0 t + \psi^0 t^2) + \varepsilon(t), \quad t \in [0, T], \tag{1}$$

is observed, where $A^0 > 0$, $\theta^0 = (\phi^0, \psi^0) \in \Phi \times \Psi = (\underline{\phi}, \overline{\phi}) \times (\underline{\psi}, \overline{\psi})$, $0 < \underline{\phi} < \overline{\phi} < \infty$, $0 < \underline{\psi} < \overline{\psi} < \infty$; $\varepsilon = \{\varepsilon(t), t \in \mathbb{R}\}$ is a stochastic process defined on the probability space $(\Omega, \mathcal{F}, P)$ and satisfying the following conditions.

A. ε is a sample-continuous stationary Gaussian process with zero mean and covariance function (c.f.) $B(t) = E\varepsilon(t)\varepsilon(0)$, having one of the properties:

(i) $B(t) = L(|t|)|t|^{-\alpha}$, $\alpha \in (0, 1)$, with non-decreasing slowly varying at infinity function L;

(ii) $B(\cdot) \in L_1(\mathbb{R})$.

Denote by A^c the closure of a set A.

Definition 1. We will call the periodogram-like estimate (PLE) of an unknown parameter $\theta^0 = (\phi^0, \psi^0)$, obtained by observation of stochastic process $X(t)$, $t \in [0, T]$, a random vector $\theta_T = (\phi_T, \psi_T) \in \Phi^c \times \Psi^c$ such that

$$Q_T(\theta_T) = \max_{\theta \in \Phi^c \times \Psi^c} Q_T(\theta), \quad Q_T(\theta) = \left| 2T^{-1} \int_0^T X(t) \exp\{i(\phi t + \psi t^2)\} dt \right|^2.$$

At the same time, we will define the amplitude A^0 estimate as $A_T = Q_T^{1/2}(\theta_T)$.

Let's formulate a theorem on the consistency of the periodogram-like estimates of the unknown chirp signal parameters.

Theorem 1. *If the process ε satisfies condition **A**, then $A_T^2 = Q_T(\theta_T) \to (A^0)^2$, $T(\phi_T - \phi^0) \to 0$, $T^2(\psi_T - \psi^0) \to 0$ a.s., as $T \to \infty$.*

To obtain an asymptotic normality result, we need a condition related to the spectral density of random noise ε.

B.

(i) The process ε that satisfies the condition **A*(i)*** has a spectral density $f(\lambda) = \widetilde{L}\left(\frac{1}{|\lambda|}\right)|\lambda|^{\alpha-1}$, where $\widetilde{L}$ is a slowly varying at infinity function, and f has the 4th spectral moment.

(ii) The spectral density of the process ε that satisfies the condition **A*(ii)*** has the 4th spectral moment.

Theorem 2. *Let conditions* ***A*** *and* ***B*** *hold. Then the random vector*

$$\left(T(A_T - A^0), T^2(\phi_T - \phi^0), T^3(\psi_T - \psi^0)\right)^* \tag{2}$$

is asymptotically normal as $T \to \infty$*, with the limiting expectation*

$$\left(A^0 a_2(\theta^0), 36a_1(\theta^0), -30a_1(\theta^0)\right)^*,$$

$$\begin{matrix} a_1(\theta^0) \\ a_2(\theta^0) \end{matrix} = \frac{1}{\sqrt{2\psi^0}}\left[\cos\left(\frac{(\phi^0)^2}{2\psi^0}\right)\left(\sqrt{\frac{\pi}{8}} - \begin{matrix} S \\ C \end{matrix}\left(\frac{\phi^0}{\sqrt{2\psi^0}}\right)\right)\right.$$
$$\left. \mp \sin\left(\frac{(\phi^0)^2}{2\psi^0}\right)\left(\sqrt{\frac{\pi}{8}} - \begin{matrix} C \\ S \end{matrix}\left(\frac{\phi^0}{\sqrt{2\psi^0}}\right)\right)\right],$$

where the functions $C(x) = \int_0^T \cos(t^2)dt$*,* $S(x) = \int_0^T \sin(t^2)dt$*,* $x \in \mathbb{R}$*.*

The limiting covariance matrix of vector (2) has the form

$$\begin{pmatrix} 4G_{22}(\theta^0) & 144G_{21}(\theta^0)/A^0 & -120G_{21}(\theta^0)/A^0 \\ 144G_{21}(\theta^0)/A^0 & 5184G_{11}(\theta^0)/(A^0)^2 & -4320G_{11}(\theta^0)/(A^0)^2 \\ -120G_{21}(\theta^0)/A^0 & -4320G_{11}(\theta^0)/(A^0)^2 & 3600G_{11}(\theta^0)/(A^0)^2 \end{pmatrix}, \tag{3}$$

where the matrix (3) has rank 2 if $G_{11}(\theta^0)G_{22}(\theta^0) - G_{21}^2(\theta^0) \neq 0$*. Here, the functions* $G_{i,j}(\theta^0)$*,* $i, j = \overline{1,2}$*, denote the elements of the limiting covariance matrix*

$$\begin{pmatrix} G_{11}(\theta^0) & G_{21}(\theta^0) \\ G_{21}(\theta^0) & G_{22}(\theta^0) \end{pmatrix}$$

of the random vector

$$\left(\int_0^T \varepsilon(t)\sin(\phi^0 t + \psi^0 t^2)dt, \int_0^T \varepsilon(t)\cos(\phi^0 t + \psi^0 t^2)dt\right)^*.$$

The proof of this theorem is based on the general approach of work [1].

1. Ivanov, O. V., Lymar, O. V. (2019). Asymptotic properties of periodogram parameter estimators for a trigonometric observation model on the plane. Theory of Probability and Mathematical Statistics, 2(101), 115-133.

Description of translation-invariant ground states for the four-state Potts-SOS model

M. M. Rahmatullaev, M. A. Rasulova

V.I.Romanovskiy Institute of Mathematics, Uzbekistan Academy of Sciences, Tashkent, Uzbekistan

mrahmatullaev@rambler.ru, m_ rasulova_ a@rambler.ru

In this work, we investigate the Potts-SOS model on the Cayley tree Γ^k of order $k \geq 2$, where the spin variable takes values in the discrete set $\Phi = \{0, 1, 2, 3\}$, and provide a complete characterization of all translation-invariant ground states corresponding to this model.

For $A \subseteq V$, a spin *configuration* σ_A on A is a function

$$\sigma_A : A \to \Phi, \qquad x \mapsto \sigma_A(x) \in \Phi \text{ for all } x \in A.$$

The set of all configurations on A is $\Omega_A = \Phi^A$. Denote $\Omega = \Omega_V$ and $\sigma = \sigma_V$.

The Potts-SOS model (see [1]) is defined by the following Hamiltonian

$$H(\sigma) = -J_P \sum_{\langle x,y \rangle \in L} \delta_{\sigma(x)\sigma(y)} - J_S \sum_{\langle x,y \rangle \in L} \mid \sigma(x) - \sigma(y) \mid, \tag{1}$$

where $J_P, J_S \in \mathbb{R}$, $\sigma \in \Omega$.

Remark 1. In (1), setting $J_P = 0$ yields the SOS model [2], while setting $J_S = 0$ leads to the Potts model [3].

We denote the following set:

$$A = \{(J_P, J_S) \in \mathbb{R}^2 : J_S \leq J_P,\, 3J_S \leq J_P\}.$$

The following theorem describes all translation-invariant ground states for the four-state Potts-SOS model on the Cayley tree of arbitrary order.

Theorem 1. *For the four-state Potts-SOS model on the Cayley tree Γ^k of order $k \geq 2$, the following assertions hold:*
1) if $(J_P, J_S) \in A$, then the configurations $\sigma(x) = i$ for all $x \in V$, $i \in \Phi$, are translation-invariant ground states;
2) if $(J_P, J_S) \in \mathbb{R}^2 \setminus A$, then no translation-invariant ground state exists.

1. Rahmatullaev M. M., Rasulova M. A. Ground states and Gibbs measures for the Potts-SOS Model with an external field on the Cayley tree. Lobachevskii Journal of Mathematics, 2024, V. 45, N. 1, 518-531.
2. Rozikov U. A. Gibbs Measures on Cayley Trees. — Singapore: World Scientific, 2013, 398 p.
3. Rozikov U. A. Gibbs Measures in Biology and Physics: The Potts Model. — Singapore: World Scientific, 2023, 368 p.

Modeling of seismic data processing using integral geometry problems on a family of broken lines

N. U. Uteuliev[1], A. K. Seidullaev[2,3], S. S. Nizamatdinova[2]
[1]Nukus State Technical University, Nukus, Uzbekistan
[2]Karakalpak State University, Nukus, Uzbekistan
[3]V. I. Romanovskiy Institute of Mathematics, Uzbekistan Academy of Sciences, Tashkent, Uzbekistan
abat1984@gmail.com

Integral geometry problems provide a mathematical framework extensively applied in seismological data processing. It involves the transformation of seismic waveforms into alternative domains to enhance analytical tractability and interpretative clarity. This approach is particularly effective for detecting and characterizing linear features within seismic datasets, which are indicative of constant ray parameters and play a critical role in subsurface imaging.

The Radon transform, commonly referred to as the slant stack, is a mathematical technique employed to reproject seismic data from the conventional time-offset domain into the $\tau - p$ domain, where τ denotes the intercept time and p represents the ray parameter [1]. This transformation facilitates the alignment of seismic traces along trajectories of constant slowness, thereby enhancing the interpretability of wavefield characteristics. It is particularly advantageous for analyzing high-resolution reflection and refraction datasets, especially those acquired from linear source geometries in horizontally layered media. The method proves effective in isolating and interpreting near-vertical reflection events, contributing to improved subsurface imaging and velocity model estimation [2].

This study addresses the inverse problem of reconstructing an unknown function $u(x, y)$, which represents the spatial distribution of a wave field from observed seismic measurements $f(x, y)$. The problem is formulated within the framework of integral geometry, specifically over a family of broken-line trajectories. An analytical relationship is derived that connects the integral transforms of the unknown wave field function $u(x, y)$ with the measured data $f(x, y)$. Furthermore, it is demonstrated that the solution can be expressed through the application of the Laplace operator to the known function $f(x, y)$, providing a pathway for efficient computational recovery.

We investigate a class of integral geometry problems characterized by the equation

$$\int\limits_{\gamma(x,y)} (y-\eta)\, u\,(\xi,\eta)\, ds = f\,(x,y)$$

where $u\,(\xi,\eta)$ is the unknown function representing the wave field distribution, and $f(x, y)$ is a known function derived from seismic measurements. The integration is performed along the curve $\gamma(x, y)$, defined by the condition:

$$\gamma\,(x,y) = \{|x-\xi| = y-\eta,\ x \in R,\ 0 < \eta \le y < H\}$$

where ds denotes the differential arc length along the curve. This formulation models the propagation of seismic waves along broken-line trajectories and serves as a foundation for reconstructing the underlying wave field from integral measurements.

Theorem 1. *Let $f(x, y)$ be a prescribed function defined on the domain $L_H = \mathbb{R} \times [0, H)$, and let $u(x, y)$ denote an unknown function that is finite on L_H and satisfies the integral equation*

$$\int\limits_{\gamma(x,y)} (y-\eta) u(\xi,\eta)\, ds = f(x,y),$$

where $\gamma(x, y)$ represents a specific curve in the domain, and ds denotes the differential arc length along this curve.

Under the assumption that $u(x,y) \in C_0^2(L_H)$ *the solution to this inverse problem is unique and can be explicitly expressed in terms of the known function* $f(x,y)$ *via the relation*

$$\Delta u(x,y) = \frac{\sqrt{2}}{4}\left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right)^2 f(x,y),$$

where $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ *denotes the Laplace operator. This result provides a direct analytical mechanism for recovering the wave field distribution from integral measurements along broken-line trajectories.*

Acknowledgements

1. Munadi S. The hyperbolic Radon transform and some of its applications in seismic data processing //Scientific Contributions Oil and Gas. 1992. V. 15. No 1. pp. 12–19.
2. Chapman C. H. Generalized Radon transforms and slant stacks //Geophysical Journal International. 1981. V. 66. No 2. pp. 445–453.

INFINITE-DIMENSIONAL DETERMINISTIC AND STOCHASTIC SYSTEM

On exponential dichotomy for differential equation with delay

A. V. Chaikovskyi
Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
andriichaikovskyi@knu.ua

Let $(X, ||\cdot||)$ be a complex Banach space, and $L(X)$ be the space of linear continuous operators in X. Consider the differential equation

$$x'(t) = Ax(t-1),\ t \in \mathbb{R}, \tag{1}$$

where $A \in L(X)$ is known and $x \in C^\infty(\mathbb{R}, X)$ is unknown.

In case of the corresponding equation without delay, it is well known that if spectrum of operator A do not intersect imaginary axis, exponential dichotomy is generated by Riss spectral decomposition. In the case of delay, the issue of exponential dichotomy has been studied by many famous scientists. The most general results belong to J.K. Hale [1,2]. In this case there are no natural spectral decomposition for A which generate exponential dichotomy.

We show that exponential dichotomy became natural if we transform the differential equation (1) to equivalent difference equation

$$x_{n+1} = Cx_n,\ n \in \mathbb{Z}, \tag{2}$$

in the Banach space $Z_1 := C^1([0,1], X)$, where $(Cx)(t) = x(1) + \int_0^1 x(s)ds,\ t \in [0,1]$.

Let us consider for equation (2) the evolution operator $T(p),\ p \in \mathbb{Z}$, which transform x_r into x_{r+p} for any $r \in \mathbb{Z}$.

Theorem 1. *If the condition $\sigma(A) \cap \{ise^{is} \mid s \in \mathbb{R}\} = \varnothing$ is satisfied, then the difference equation (2), admits an exponential dichotomy: there exist subspaces Z_+, Z_- of the space Z_1 such that:*

1) the direct sum of Z_- and Z_+ is equal to the space Z_1;

2) for the projector P_- onto the subspace Z_- the following estimate holds

$$\exists L > 0\ \exists q \in (0,1)\ \forall p \geq 0 :\ ||T(p)P_-||_1 \leq Lq^p;$$

3) for the projector P_+ onto the subspace Z_+ the following estimate holds

$$\exists L > 0\ \exists q \in (0,1)\ \forall p \geq 0 :\ ||T(-p)P_+||_1 \leq Lq^p.$$

Note that the obtained result strengthens the corresponding Hale's result. In addition, an explicit integral representation for the operators P_-, P_+ is obtained.

This result could be generated to a case of several delays

$$x'(t) = \sum_{k=1}^{m} A_k x(t-k),\ t \in \mathbb{R},$$

where $\{A_k\ :\ 1 \leq k \leq m\} \subset L(X)$.

1. Hale J.K., Zhang W. On uniformity of exponential dichotomies for delay equations. J. Differential Equations, 2004, 204, P. 1 – 4.
2. Hale J.K. Theory of Functional Differential Equations, Springer, New York, 1977.

Finite-time Stabilization of Parabolic SPDEs under Cyberattacks

Nidhi Shukla[1]

[1]Indian Institute of Technology Roorkee, Uttarakhand, India

n_shukla@as.iitr.ac.in

Abstract

This paper presents a comprehensive framework for the robust and reliable non-fragile control design for second-order stochastic PDE system subject to parametric uncertainties, external disturbances and deception attacks for the finite-time stability case. By employing Lyapunov stability theory and transforming the resulting conditions into nonlinear matrix inequalities, we establish computational procedures for controller synthesis that guarantee finite-time boundedness while achieving prescribed performance attenuation levels. The proposed approach effectively balances theoretical rigour with practical implementation, addressing a critical need in modern stochastic control systems design. A numerical example is provided to demonstrate the effectiveness of the proposed approach.

Acknowledgements: Nidhi Shukla extends her gratitude to the Ministry of Education, Government of India (Prime Minister's Research Fellowship, PMRF ID: 2802880) for the financial assistance provided to carry out her research.

References

[1] J. Zhang, J. Song, J. Li, F. Han, and H. Zhang, "Observer-based non-fragile H_∞-consensus control for multi-agent systems under deception attacks," *Int. J. Syst. Sci.*, vol. 52, no. 6, pp. 1223-1236, 2021.

[2] X. Wang, J. H. Park, and H. Li, "Fuzzy secure event-triggered control for networked nonlinear systems under DoS and deception attacks," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 53, no. 8, pp. 4165-4175, 2023.

[3] L. Shao, W. Sun, and L. Ding, "Finite-time adaptive control of output constrained nonlinear systems under deception attacks," *Nonlinear Dyn.*, vol. 112, no. 19, pp. 17273-17290, 2024.

[4] J. Zhao and G.-H. Yang, "Observer-Based Finite-Time Fuzzy Adaptive Resilient Control for Uncertain Nonlinear Systems Against Deception Attacks and Unknown Dead Zones," *IEEE Trans. Autom. Sci. Eng.*, vol. 22, pp. 8762-8772, 2025, doi: 10.1109/TASE.

[5] Nidhi Shukla, N. Keerthana, R. Sakthivel, V.T. Elayabharath, Jaydev Dabas, "Observer-reliant event-triggered security control design for stochastic third-order PDE systems with multiple attacks," *European Journal of Control*, vol. 82, 101186, Jan. 2025. https://doi.org/10.1016/j.ejcon.2025.101186

Functional limit theorems for perturbed random walks

A. Pilipenko[1]

[1]Institute of Mathematics of the Ukrainian National Academy of Sciences, Kyiv, Ukraine

pilipenko.ay@gmail.com

A random sequence $S_\xi(n) := S_\xi(0) + \sum_{k=1}^{n} \xi_k, n \geq 1$, is called a random walk, where (ξ_n) are independent identically distributed random variables that are independent of $S_\xi(0)$.

Let $m \in \mathbb{N}_0$, ξ and $\eta_i, |i| \leq m$, be integer-valued random variables. Consider a Markov chain $(X(n))$ with transition probabilities

$$p_{i,j} = \begin{cases} \mathrm{P}(\xi = j - i), & |i| > m; \\ \mathrm{P}(\eta_j = j - i), & |i| \leq m, \end{cases} \tag{1}$$

which is a perturbation of a random walk S_ξ. Formula (1) means that jumps outside the set $\{-m, \ldots, m\}$ have distribution ξ and jumps from a point $i, |i| \leq m$ has distribution η_i. To avoid certain trivial cases we will always assume that all states in membrane communicate and the random walk can exit the membrane with probability 1.

It is well known that if ξ is a zero mean random variable with finite variance, then Donsker's scaling limit of S_ξ s a Brownian motion. We discuss scaling limits of the perturbed random walk X.

Theorem 1. *Assume that* $\mathrm{E}\xi = 0, \sigma^2 = \mathrm{Var}(\xi) \in (0, \infty), \mathrm{E}|\eta_i| < \infty, |i| \leq m$. *Then sequence of stochastic processes* $(\frac{X([nt])}{\sigma\sqrt{n}}, t \geq 0)_{n\geq 1}$ *converges in distribution to a skew Brownian motion as* $n \to \infty$.

We also consider scaling limits of X if ξ and η_i have regularly varying tails.

1. Iksanov A., Marynych A., Pilipenko A., Samoilenko I. Locally Perturbed Random Walks. — Birkhauser, 2025, 224 p.

Constructing stochastic flows of kernels

G.V. Riabov

[1]Institute of Mathematics of NAS of Ukraine, Kyiv, Ukraine

ryabov.george@gmail.com

Let M be a locally compact separable metric space. Let $\mathsf{P}^{(n)}$ be a consistent sequence of Feller transition functions on M^n. We prove that there exists a stochastic flow of kernels on M such that

1. distributions of $n-$point motions of a flow are determined by $\mathsf{P}^{(n)}$;
2. there exists a single idempotent measurable presenation $\mathfrak{p}$ of distributions of kernels from a flow;
3. a flow is invariant with respect to $\mathfrak{p}$.

1. Riabov G. Constructing stochastic flows of kernels. Theory of Stochastic Processes, Vol. 29 (45), no. 1, 2025, pp. 90 –117.

A numerical method for a fractional-order evolution equation in a Banach space

V. L. Ryabichev[1], **N. V. Mayko**[2]
[1]Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
[2]Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
ryabichev@knu.ua, mayko@knu.ua

We consider an initial-value problem for an abstract evolution equation [1]:

$$\partial_t u + \partial_t^{-\alpha} A u = f(t), \quad t \in (0;T],$$
$$u(0) = u_0,$$

where A is a strongly positive operator in a Banach space E, $f : [0;T] \to E$ is a given vector-valued function, $u_0 \in E$ is a given vector, $u : [0;T] \to E$ is a sought solution, $\partial_t = \frac{d}{dt}$, and

$$(\partial_t^{-\alpha} u)(t) = \begin{cases} \partial_t \displaystyle\int_0^t \frac{(t-s)^\alpha}{\Gamma(1+\alpha)} u(s)\, ds & \text{if} \quad -1 < \alpha \leqslant 0; \\ \displaystyle\int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} u(s)\, ds & \text{if} \ \ 0 < \alpha < 1. \end{cases}$$

In many applications, A represents a linear second-order differential operator involving partial derivatives with respect to spatial variables and time-independent coefficients.

We recall the definition of a strongly positive operator [2].

Definition 1. Let E be a Banach space. A closed linear operator $A : D(A) \to E$ with dense domain $D(A) \subset E$ and resolvent set $\rho(A) \subset \mathbb{C}$ is called *strongly positive* if there exist constants $\varphi \in (0; \pi/2)$, $\gamma > 0$, and $L > 0$ such that $\left\|(zI - A)^{-1}\right\| \leqslant \dfrac{L}{1+|z|}$ for all $z \in \Sigma$, where I denotes the identity operator, and $\Sigma = \left\{z \in \mathbb{C} \,|\, \varphi \leqslant |\arg z| \leqslant \pi\right\} \bigcup \left\{z \in \mathbb{C} \,|\, |z| \leqslant \gamma\right\} \subset \rho(A)$.

The solution $u(t)$ can be formally presented in the form:

$$u(t) = U(t) u_0 + \int_0^t U(t-s) f(s)\, ds,$$

where $U(t) = \sum\limits_{n=0}^{\infty} p_n^{(\alpha)}(t^{1+\alpha}) Q^n$ is a solving operator, $Q = A(I+A)^{-1}$ is the Cayley transform of the operator A, and $p_n^{(\alpha)}(t^{1+\alpha})$, $n = 0, 1, \ldots$, are the Laguerre–Cayley functions:

$$p_n^{(\alpha)}(t^{1+\alpha}) = \sum_{r=0}^{n-1} \frac{(-1)^{r+1} C_{n-1}^r t^{(r+1)(\alpha+1)}}{\Gamma\big(1 + (r+1)(\alpha+1)\big)}, \; n \in \mathbb{N}, \quad p_0^{(\alpha)}(t^{\alpha+1}) \equiv 1.$$

Next, we approximate $u(t)$ as follows:

$$u_N(t) = U_N(t) u_0 + \int_0^t U_N(s) f(t-s)\, ds, \quad U_N(t) = \sum_{n=0}^{N} p_n^{(\alpha)}(t^{1+\alpha}) Q^n. \tag{1}$$

We study the accuracy of $u_N(t)$ under certain conditions regarding the input data u_0 and $f(t)$. We also assume that the Laguerre–Cayley functions $p_n^{(\alpha)}(t^{1+\alpha})$ satisfy the inequality

$$\left|p_n^{\alpha}(t^{1+\alpha})\right| \leqslant C(t) n^\gamma, \tag{2}$$

where $\gamma \in \mathbb{R}$ and $C(t) > 0$ is independent of n. The assumption is based on the substantial number of evaluations carried out using the computer algebra system Maple.

For finitely smooth (in some sense) u_0 and $f(t)$, we obtain the following result [3].

Theorem 1. *Let the Laguerre–Cayley functions* $p_n^{(\alpha)}(t^{1+\alpha})$ *satisfy condition (2) with* $\int_0^t C^2(s)\,ds < \infty$, $u_0 \in D(A^\sigma)$ *with* $\sigma > \max\{0; 2(\gamma+1)\}$, *and let the right-hand side* $f(t)$ *meet the conditions:* $f(t) \in D(A^\sigma), \sigma > \max\{0; 2(\gamma+1)\}, \int_0^t \|A^\sigma f(s)\|^2\,ds < \infty$.

Then, the Cayley transform method (2) is free from accuracy saturation, and for all ε *such that* $0 < \varepsilon < \min\{1; \sigma - 2(\gamma+1)\}$, *and all integers* N *such that* $N+1 \geqslant \sigma$, *the following estimate holds true:*

$$\|u(t) - u_N(t)\| \leqslant \frac{M(t)}{N^{\frac{\sigma-\varepsilon}{2}-\gamma-1}} \left\{ \|A^\sigma u_0\| + \left[\int_0^t \|A^\sigma f(s)\|^2\,ds \right]^{1/2} \right\},$$

where $M(t) > 0$ *is independent of* N.

Thus, the Cayley transform method (2) has a power-type convergence rate (with respect to the parameter N) close to $O\Big(\frac{1}{N^{\sigma/2-\gamma-1}}\Big)$. This estimate indicates that the method does not exhibit accuracy saturation: its accuracy automatically depends on the regularity of the right-hand side $f(t)$ and the initial vector u_0. The parameter σ reflects the smoothness of the input data whereas γ is related to the properties of the Laguerre–Cayley polynomials, the study of which remains an active area of research. The case of infinitely smooth input data u_0 and $f(t)$ will be addressed in [4].

1. McLean W., Thomée V. Numerical solution via Laplace transforms of a fractional order evolution equation. J. Integral Equ. Appl., 2010, volume 22, No. 1, p. 57-94. https://doi.org/10.1216/JIE-2010-22-1-57
2. Gavrilyuk I. P., Makarov V. L. The Cayley transform and the solution of an initial value problem for a first order differential equation with an unbounded operator coefficient in a Hilbert space. Numer. Funct. Anal. Optim., 1994, vol. 15, No. 5–6, p. 583–598. https://doi.org/10.1080/01630569408816582
3. Makarov V. L., Mayko N. V., Ryabichev V. L. A numerical method without accuracy saturation for a fractional-order evolution equation in a Banach space. J. Appl. Numer. Anal., 2025, vol. 3, p. 102-112. https://doi.org/10.30970/ana.2025.3.102
4. Makarov V. L., Mayko N. V., Ryabichev V. L. An exponentially convergent numerical method for a fractional-order evolution equation in a Banach space. Cybern. Syst. Anal., 2026 (to appear).